OBERWOLFACH SEMINAR PROGRAMME
METRIC TOPOLOGY OF ASPHERICAL SPACES
OCTOBER 20–25, 2024

1. Organisational matters

1.1. Organisers.

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2. The seminar

2.1. Overview. Metric topology lies on the border between algebraic topology and metric geometry. It studies how topological complexity and geometric complexity relate to each other.

Classical examples of this paradigm are Gromov's and Thurston's proof of Mostow rigidity via simplicial volume or Gromov's bound of the minimal volume by the simplicial volume. A recent example is the breakthrough result of Chambers–Manin–Weinberger on Lipschitz bounds for nullhomotopies of nullhomotopic Lipschitz maps and its application to quantitive bordism theory. Other recent developments in metric topology include results on macroscopic scalar curvature, filling inequalities, and homological gradient bounds from geometric conditions.

In this seminar, we will outline basic techniques of metric topology and guide the participants to recent developments of this research area. We will put a special emphasis on aspherical spaces for which the fundamental groups governs the topology.

2.2. Detailed description. Metric topology combines algebraic topology with metric data. We will focus on three major themes:

(a) Refining and metrically enriching homology.
(b) Topological invariants and metric notions of largeness.
(c) Topological and metric filling problems.

(a) The singular chain complex can be refined by different norms. Most notably, there is the simplicial and flat norm. The simplicial volume of a closed manifold is the simplicial norm of the fundamental class; it is a measure of the topological complexity. Its interplay with the flat norm leads to

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interesting connections between volume and topology [5]. Ergodic-theoretic variants of the simplicial norm will be considered. They provide a link between the geometry and homological invariants like homology gradients, cost, and ℓ^2-Betti numbers [4, 16, 17, 20].

(b) A basic metric notion of largeness is volume. Gromov's main inequality bounding the simplicial volume by the Riemannian volume [5] serves as a prototype result. We discuss a simplified proof based on the ergodic-theoretic variant of simplicial volume introduced in (a) and on the classical tool of nerves of open covers. We adapt the approach via nerves and classifying spaces to various metric and ergodic theoretic settings [11, 14, 15, 18]. In particular, we will present the corresponding applications to vanishing results of homological gradient invariants. From there one could discuss generalizations of Gromov's main inequality obtained [3, 9]. Other (related) metric notions of largeness are the filling radius and the Uryson width [10, 19]. We address Gromov's and Guth's macroscopic perspective that enables interesting analogies with differential-geometric conjectures and results [1, 2, 8]. For example, Papasoglu's recent progress on the Uryson width [19] ultimately relies on a metric and coarse analogue of Schoen–Yau's minimal surface technique.

(c) It is a basic problem in topology to decide wether a map from a sphere S^n to a space can be extended to the ball D^{n+1}. Metric variations of this extension problem can be formulated in the context of normed chain complexes or metric spaces where one asks for a Lipschitz or volume control of the maps. We give a unified perspective on these filling problems, which encompass the question of the comparison between bounded and ordinary cohomology, isoperimetric inequalities, higher filling functions, the Lipschitz extension problem [13], and quantitive bordism theory [12]. Obviously, this would be too much to cover in detail. Thus the focus will be on the perspective and how variations of the extension problem lead to very different geometric questions.

This seminar will be accessible to all graduate students and postdocs with a basic understanding of classical algebraic topology (fundamental groups, covering theory, ordinary homology, Poincaré duality, CW-complexes), group actions, and geometry (manifolds, Riemannian volume). The techniques and results could be of interest to participants working in geometric topology, metric geometry, or geometric group theory.

2.3. **Structure.** We will follow the traditional structure of Oberwolfach seminars:

- Morning session: There will be two lectures of the following form, separated by a longer coffee/tea break:

  45 minutes lecture + 5 minutes break + 45 minutes lecture

  Each lecture will be accompanied by a small worksheet, containing exercises of various difficulty levels (quick checks, examples, more challenging problems).
- Break after lunch: Time for the participants to review the lecture material and to start working on the worksheet.

- Afternoon session: Discussion of questions concerning the lectures, discussion of the examples and exercises from the worksheets.

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