Reduction of Arithmetic Varieties

Oberwolfach Seminar 2024

General information

This workshop is devoted to some of the most important varieties of a global nature which arise in the Langlands program: Shimura varieties and moduli spaces of G-shtukas. For each of these categories, there are different local directions which have been pursued.
On one hand there are the local models, which provide links to objects in the geometric Langlands program such as Beilinson-Drinfeld Grassmannians, and serve to elucidate local behavior of nearby cycles sheaves and singularities of the global objects.
On the other hand, there are local analogues of the global objects: the local Shimura varieties, and the moduli spaces of local G-shtukas.
These provide sources for uniformization theorems and provide geometric spaces whose cohomology groups are useful in realizing aspects of local Langlands correspondences, analogous to the role the global objects play in realizing certain global Langlands correspondences.
These lectures will discuss both local directions, for both types of global objects, and will highlight the recent developments and applications of Scholze's ideas in p-adic geometry. We will also point out some open problems and fruitful directions for future research. The workshop will include problems.

Eva Viehmann: Moduli spaces global and local G-shtukas

The main topic is local and global moduli spaces in the function field case. More precisely, moduli of global G-shtukas and the corresponding local models are explained. Then the Kottwitz poset B(G) and moduli of local shtukas are introduced. Finally, the relation between the local and the global moduli space is stated.

Thomas J. Haines: Scheme-theoretic local models

These lectures will give definitions and basic properties of the scheme-theoretic local models in the sense of Rapoport—Zink and Pappas—Zhu, highlighting the use of Beilinson—Drinfeld Grassmannians as ind-schemes over mixed characteristic DVRs, and the local model diagrams linking back to global Shimura varieties. The focus will then be on applications to nearby cycles and singularities of certain integral models of Shimura varieties.

Prerequisites:
linear algebraic groups and (parahoric) group schemes, ind-schemes, familiarity with some aspects of Bruhat-Tits buildings, as well as nearby cycles sheaves, affine Grassmannians, and Schubert varieties

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Timo Richarz: p-adic local models

The lecture studies the p-adic theory of local models developed by Scholze and their relation to the schematic theory from Haines' lecture. The focus lies on sheaf-theoretic techniques such as nearby cycles that are used to access the geometry of (p-adic) local models. We make simplifying assumptions whenever suitable and illustrate the theory in examples.

Prerequisites:
linear algebraic groups, torsors, p-adic geometry including diamonds and their étale cohomology, period rings

Torsten Wedhorn: Local Shimura varieties

In this lecture we will consider local Shimura varieties and their relation to global Shimura varieties and to Rapoport-Zink spaces. In general, they will be constructed as diamonds in the sense of Scholze. Definitions, statements, and constructions will be based on examples.

Prerequisites:
Theory of linear algebraic groups, abelian varieties and p-divisible groups, p-adic geometry including theory of diamonds