Oberwolfach Seminar

Tensor triangular geometry and interactions

Organizers: Natàlia Castellana, Henning Krause, Greg Stevenson

The purpose of the seminar is to give an introduction to tensor triangular geometry and recent developments in equivariant homotopy theory and the representation theory of finite groups. Tensor triangular geometry is a general framework for studying triangulated categories equipped with a tensor product. Viewing the additive structure of the category as resembling an abelian group, and the symmetric monoidal structure as a commutative multiplication, one can consider the prime ideal spectrum with the Zariski topology in analogy with commutative algebra. This gives rise to a rich theory and allows ideas from algebraic geometry, in particular descent, to be applied to representation theory and homotopy theory.

There will be three series of lectures. One of them will focus on giving an introduction to the subject prioritizing the lattice-theoretic perspective with a view toward discussing novel support theories for compactly generated categories and descent. Another will approach the subject via the action of the endomorphism ring of the unit and explain the structure of modular representations of finite groups and recent progress on permutation modules. The third series will present the passage from the classification theorem for finite spectra to our current understanding of equivariant spectra using geometric fixed points and descent.

In fact, each of the categories mentioned above is not just a triangulated category. They all arise as homotopy categories of symmetric monoidal ∞-categories, and this is the main source of examples. This higher homotopy coherent structure turns out to be crucial in formulating the correct descent statements which express how these categories are glued together from local pieces. That said, the fundamental aspects of the theory are purely triangulated and no knowledge of stable ∞-categories will be assumed (but rather we hope to impart some feeling of familiarity).

Reading list

- Paul Balmer, A guide to tensor-triangular classification, in *Handbook of homotopy theory*, 145–162, CRC Press/Chapman Hall Handb. Math. Ser., CRC Press, 2020
- Paul Balmer, Tensor triangular geometry, in *Proceedings of the International Congress of Mathematicians. Volume II*, 85–112, Hindustan Book Agency, 2010.
- David J. Benson, Srikanth B. Iyengar and Henning Krause, *Representations of finite groups: local cohomology and support*, Oberwolfach Seminars, 43, Birkhäuser/Springer Basel AG, 2012
- Mike Hill, *Equivariant stable homotopy theory*, in *Handbook of homotopy theory*, 699–756, CRC Press-Chapman Hall Handb. Math. Ser., CRC Press, 2020
- Amnon Neeman, *Triangulated categories*, Ann. Math. Stud., 148, Princeton Univ. Press, 2001
- Greg Stevenson, A tour of support theory for triangulated categories through tensor triangular geometry, in *Building bridges between algebra and topology*, 63–101, Adv. Courses Math. CRM Barcelona, Birkhäuser/Springer, 2018