OBERWOLFACH SEMINAR: PHASE TRANSITIONS AND TURBULENCE INCLUDING FLUIDS
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How do we model complicated phenomena including fluids like phase transition and turbulence by using differential equations? How do we analyze such equations which are often challenging?

The weak-long "Oberwolfach Seminar 2543b" will be devoted to such questions. The target audience is PhD students or post-doctoral researchers wishing to be quickly immersed in a modern, active research area. The number of participants is limited to 24; priority will be given to young, motivated researchers.

M.-H. Giga and Y. Giga will begin with several axiomatic ways of derivation of the Navier-Stokes-Korteweg type equation, which is a very classical model to describe liquid-vapor phase transition. Some fundamental analysis will be also given.

C. Liu will give lectures on energetic variational approaches for active materials and reactive fluids and their applications in complex fluids. Active materials and reactive fluids play pivotal roles in numerous biological and physical applications. These materials encompass those capable of converting and transducing energy of different types, particularly between mechanical energy and various forms of chemical energy. They are ubiquitous in applications and have gained increasing significance in fields such as clean energy, environmental sciences, and medical and biological systems. In these lectures, C. Liu will present a comprehensive and unified framework to model these specific types of active fluids. This extension is the extension of the classical energetic variational approaches for mechanical systems. These energetic variational approaches can be employed to a broad spectrum of chemical reaction kinetics coupling with mechanical processes.

A. Schlömerkemper will give lectures on the mathematics of materials that show magnetic as well as elastic behaviour. While elastic materials are naturally modelled in the reference configuration, the typical starting point for magnetic materials is the current configuration. A. Schlömerkemper will present different approaches to the modelling and analysis of the coupled system for different materials including magneto-viscoelastic fluids.

T. Yoneda will give lectures on tubulence. Scale decomposition is crucial in the study of turbulence (meteorology). For example, it is important (for all people) to predict large scale weather phenomena such as typhoons and El Nino phenomena, and in these cases it is quite natural to assume that the large scale fluid motions are little affected by small scale motions. T. Yoneda will mathematically explain the recent turbulence study in terms of scale decomposition (i.e. Littlewood-Paley theory). If time allows, he will also explain a practical filter theory and give a short machine learning exercise. In this case we will use Google Collaboratory, so please have a Google account before his lecture.

Recommended reading for Giga^2's lectures

[1] M.-H. Giga, A. Kirshtein and C. Liu, Variational modeling and complex fluids. *Handbook of mathematical analysis in mechanics of viscous fluids* (Y. Giga and A. Novotný eds.), 73–113. *Springer, Cham,* 2018.

[2] J. Málek and V. Průša, Derivation of equations for continuum mechanics and thermodynamics of fluids. *Handbook of mathematical analysis in mechanics of viscous fluids* (Y. Giga and A. Novotný eds.), 3–72. *Springer, Cham,* 2018. [Section 4.3]

[3] J. Prüss and G. Simonett, Moving interfaces and quasilinear parabolic evolution equations. Monogr. Math. **105**, *Birkhäuser/Springer, Cham,* 2016. [Chapter 1]

Recommended reading for Liu's lectures

[1] M.-H. Giga, A. Kirshtein and C. Liu, Variational modeling and complex fluids. *Handbook of mathematical analysis in mechanics of viscous fluids* (Y. Giga and A. Novotný eds.), 73–113. *Springer, Cham,* 2018.

[2] Y. Wang, C. Liu, P. Liu and B. Eisenberg, Field theory of reaction-diffusion: law of mass action with an energetic variational approach. *Phys. Rev. E* **102** (2020), 062147.

[3] Y. Wang, T.-F. Zhang and C. Liu, A two species micro-macro model of wormlike micellar solutions and its maximum entropy closure approximations: An energetic variational approach. *J. Non-Newton. Fluid Mech.* **293** (2021), 104559.

[4] F. De Anna, C. Liu, A. Schlömerkemper and J.-E. Sulzbach, Temperature dependent extensions of the Cahn-Hilliard equation. *Nonlinear Anal. Real World Appl.* **77** (2024), 104056.

[5] D. Kondepudi and I. Prigogine, Modern thermodynamics: from heat engines to dissipative structures. *John Wiley & Sons,* 2014.

[6] L. Onsager, Reciprocal Relations in Irreversible Processes. I. *Phys. Rev.* **37** (1931), 405.

[7] L. Onsager, Reciprocal Relations in Irreversible Processes. II. *Phys. Rev.* **38** (1931), 2265.

Recommended reading for Schlömerkemper's lectures

[1] B. Benešová, Š. Nečasová, J. Scherz and A. Schlömerkemper, A variational approach to the modeling of compressible magnetoelastic materials. Preprint, available at `https://arxiv.org/abs/2410.15196`

[2] H. Garcke, P. Knopf, S. Mitra and A. Schlömerkemper, Strong well-posedness, stability and optimal control theory for a mathematical model for magneto-viscoelastic fluids. *Calc. Var. Partial Differential Equations* **61** (2022), 179.

[3] M. Kalousek, S. Mitra and A. Schlömerkemper, Existence of weak solutions to a diffuse interface model involving magnetic fluids with unmatched densities. *Nonlinear Differ. Equ. Appl.* **30** (2023), 52.

[4] B. Benešová, J. Forster, C. Liu and A. Schlömerkemper, Existence of weak solutions to an evolutionary model for magnetoelasticity. *SIAM J. Math. Anal.* **50** (2018), 1200–1236.

Recommended reading for Yoneda's lectures

[1] T. Yoneda, Machine learning code, available at `https://github.com/tsuyoshi-yoneda-math/reservoir_realtimefilter`

[2] T. Yoneda, S. Goto and T. Tsuruhashi, Mathematical reformulation of the Kolmogorov-Richardson energy cascade in terms of vortex stretching. *Nonlinearity* **35** (2022), 1380–1401.

[3] I.-J. Jeong and T. Yoneda, Quasi-streamwise vortices and enhanced dissipation for the incompressible 3D Navier-Stokes equations. *Proc. Amer. Math. Soc.* **150** (2022), 1279–1286.

[4] I.-J. Jeong and T. Yoneda, Vortex stretching and enhanced dissipation for the incompressible 3D Navier-Stokes equations. *Math. Ann.* **380** (2021), 2041–2072.