Mathematisches Forschungsinstitut Oberwolfach

Member of Leibniz Leibniz Association

Simons Visiting Professors

Activity Reports 2014

REPORT ON A RESEARCH PROJECT SUPPORTED BY SIMONS VISITING PROFESSORSHIP

SUMIO YAMADA

F. Klein, based on observations by Beltrami, modeled the hyperbolic plane on the open unit disc equipped by a distance function $d(x, y)$ defined as the cross ratio among the four points $b(y, x), x, y, b(x, y)$ where $b(x, y)$ is the boundary point of the disc lying on the ray starting at x passing through y. When the disc was replaced by convex bodies Ω in $\mathbb{R}^n$, the resulting metric is called Hilbert metric.

In a collaborative work (*The Funk and Hilbert geometries for spaces of constant curvature* (2013)) with Athanase Papadopoulos, the author has looked at cross ratio in constant curvature spaces, S^n and $\mathbb{H}^n$. Each spaces can be realized as the unit sphere in the vector space $\mathbb{R}^{n+1}$, in the sense that the former is the set of points/vectors $\{Q_E(x, x) := x_0^2 + \cdots + x_n^2 = 1\}$ and the latter defined by $\{Q_L(x, x) := -x_0^2 + \cdots + x_n^2 = -1\}$, We identified the cross ratio using the sine function for the sphere, and the hyperbolic sine function for the hyperbolic space. Those quantities are shown to be projective invariants, in the sense that they are realized in the constant curvature spaces using the respective distance function and the trigonometric functions, and the realizations are canonically defined by the projection map in $\mathbb{R}^{n+1}$ with respect to the origin, which gives natural correspondences with the flat space $\mathbb{R}^n$, where the classical projective geometry is modeled.

These observations had led us in several further directions of research. The first is to generalize the cross ratio to other constant curvature spaces using the projective models. Athanase Papadopoulos and the author have undertaken the project, and have started in looking into the representation of cross ratio with respect to the geometric quantities associated with de Sitter spaces, anti-de Sitter spaces, and Minkowski spaces. Such representations would induce generalizations of Hilbert metric defined on convex bodies in those non-flat spaces. We have obtained some preliminary results so far.

The second direction is generalizing cross ratio on Teichmüller spaces. In fact, the Teichmüller space of a surface can be imbedded in an infinite dimensional open cone as was first done by Thurston. He also defines the intersection number, which can be regarded as a restriction of a quadratic form defined on the open cone. There are several further indications that the existing knowledge of the Teichmüller theory can be transcribed into the language of convex geometry in the infinite dimensional cone. We have discussed the issues concerning this perspective, and plan to pursue this viewpoint for further investigations.

Date: March 5, 2014.

Acknowledgement: This report is written to document the research activities undertaken by the author while he was attending MFO meeting "New Trends in Teichmüller Theory and Mapping Class Groups" (9 February–15 February 2014) and visiting Institut de Recherche Mathématique Avancée/ Université de Strasbourg hosted by Prof. Athanase Papadopoulos over the following week (16 February–23 February 2014.) This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Department of Mathematics, Gakushuin University, Tokyo 171-8588, Japan
E-mail address: yamada@math.gakushuin.ac.jp

SCIENTIFIC ACTIVITY REPORT

HUGH THOMAS

I arrived in Germany on Sunday, February 9, and went to Bochum. On Monday, I attended the conference "New perspectives in hyperplane and reflection arrangements" in Bochum. This was a one-day conference, with a highlight being a talk by Terao. A collaborator of mine, Christian Stump, was also speaking. We discussed our project (joint also with Nathan Williams). Let Q be a Dynkin quiver, and W its corresponding Weyl group. Some eight years ago, Colin Ingalls and I showed that the torsion classes for the path algebra kQ correspond bijectively to a certain subset of W, the c-sortable elements, which had been introduced earlier by Nathan Reading. It is natural to broaden the question, asking about (generating, separated) aisles in the derived category of kQ. We believe that the correct approach to a combinatorial description of these aisles is to extend the notion of c-sortability to the corresponding Artin group. This raises questions in Artin group theory which seem to be of independent interest.

On Tuesday, February 11, I went to Bielefeld. I gave a talk in the afternoon, titled "Cofinite quotient-closed subcategories of quiver representations (plus...)." My talk reported on a project with Steffen Oppermann and Idun Reiten, arXiv:1205.3268. We study the (cofinite) quotient-closed subcategories of the representations of a quiver. Such subcategories are a generalization of the notion of torsion class. They have hardly been studied at all, but, rather surprisingly, it turns out that they are naturally in bijection with the elements of the Weyl group associated to Q, which Drew Armstrong and I had conjectured in 2007. The afternoon also included talks by Osamu Iyama and Idun Reiten.

While in Bielefeld, I worked primarily with Iyama and Reiten on a joint project of ours, together with Nathan Reading and Gordana Todorov, which began at the MSRI conference on cluster algebras in October 2012, continued for the rest of that term at MSRI, and then at the Oberwolfach workshop on cluster algebras in December 2013. Let Q be a Dynkin quiver. Let Λ_Q be the corresponding preprojective algebra. It has kQ as a quotient. The functorially finite torsion classes for Λ_Q correspond to elements of W. The quotient from Λ_Q to kQ naturally gives rise to a map from functorially finite torsion classes for Λ_Q to (functorially finite) torsion classes for kQ. According to the correspondences already mentioned, this amounts to a map from W to the c-sortable elements of W. Such a map was already known: the lattice quotient from weak order to the Cambrian lattice, defined by Reading. The initial step of our project was to establish that the algbraic and the lattice-theoretic maps agree. That much, we had accomplished at MSRI. Our goal is to extend both sides of the correspondence, studying quotient algebras which sit between the preprojective algebra and the path algebra, and other lattice quotients on weak order.

On Friday, I gave another talk in Bielefeld, titled "Combinatorics of AR-translation in finite type cluster categories, and analogues." This reported on another aspect of my project with Stump and Williams. Let Q be a Dynkin quiver. The torsion classes for kQ can be naturally ordered by inclusion; this produces the Cambrian lattice mentioned above. A feature of this partial order which has not been focussed on is that there is a natural labelling of the edges in its Hasse diagram by positive roots. This labelling turns up in a surprising number of different connections, for example in the context of cluster algebras (where it indicates, up to a sign, the c-vector of the cluster variable that is being mutated) or in my work with Christophe Hohlweg and Carsten Lange, where we show that the Hasse diagram of this poset can be realized as the 1-skeleton of a polytope, with the labels indicating the direction (though not the length) of the edges.

It is well-known that the Hasse diagram of this poset, thought of as an unoriented graph, posesses a certain symmetry induced by the AR-translation. The new feature presented in this talk was the fact that this symmetry can be accomplished by a certain walk in the Hasse diagram. This give a kind of factorization of the AR-translation, which is reminiscent of the factorizations into maximal green sequences which are relevant for calculating Donaldson-Thomas invariants.

On Sunday I went to Oberwolfach. While at Oberwolfach, I continued to work with Reiten and Iyama. I was also approached by Yuya Mizuno, who recently finished his Ph.D. under the direction of Iyama, who wanted to talk about some aspects of my paper with Oppermann and Reiten mentioned above. We are now working together to solve a question which was left open in that paper. As I discussed above, cofinite quotient-closed subcategories for kQ correspond to elements of W, and torsion classes are a special kind of quotient-closed subcategory. The question, then, is to identify which elements of W correspond to torsion classes. In my paper with Oppermann and Reiten, we gave a conjectural answer, which Mizuno and I hope to prove correct.

On Friday, I spoke at Oberwolfach, on the paper with Oppermann and Reiten. I emphasized an aspect which I had not touched on in Bielefeld, namely the link to the ⅃-diagrams of Postnikov. On Saturday, February 22, I flew back to Canada.

Acknowledgements. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I am very grateful for the opportunity to have visited Bielefeld in advance of the Oberwolfach workshop; I feel certain that much less would have been accomplished on my project with Reiten and Iyama if we had not also had the opportunity to work together intensively prior to the workshop.

Report to the Simons Foundation
Bill Helton UC San Diego April 2014

I certainly wish to think the Simons foundation for supporting my trip to Oberwolfach and to Konstanz. The conference was wonderful and the trip to Konstanz initiated a collaboration with my host and also other discussions enabled me to learn about several recent developments.

The conference was an eclectic mix of Operator Theory and real algebraic geometry RAG with some discussion of control theory mixed in. The odd thing about this is it is suited my interests very well. For example, I do not see the real algebraic geometry people often and was surprised to see how much progress they've made on the Helton-Vinnikov and the Helton-Nie conjectures. Also I was able to meet and talk a good bit to Thom who is in the Von Neumann algebra community but who also knows RAG. A young engineer who was there, Amir Ali Amadi, had some clever and I think quite valuable new approximate algorithm for solving RAG problems. These few samples illustrate many valuable conversations and talks.

At Konstantz I talked a great deal with my host Markus Schweighoffer:

Markus at
Oberwolfach

who is interested in solving classical combinatorial optimization problems using linear matrix inequalities. Klep, McCullough and I had some notes on a slightly tangential topic which we were never really able to carry too far, one reason being we did not understand the classics well enough to have a good sense of direction. Markus is familiar with the classics, so we are trying to obtain sharp estimates. Now we are making steady progress and keeping fingers crossed that the final steps we need fall in place.

Also at Konstanz, Claus Scheiderer and his students showed me basic ideas behind his considerable progress on my conjectures.

In summary the trip was very rewarding: one of my best trips in recent years.

Thanks again to the Simons foundation for support and please congratulate Jim for me on being elected to the National Academy of Sciences,

Bill Helton

SCIENTIFIC ACTIVITY REPORT

BRUCE REZNICK

I am pleased to report on my activities as a Simons Visiting Professor at the University of Konstanz and at MFO.

I arrived in Konstanz (by train, from the Zurich Airport) at noon on Sunday 30 March, 2014 and stayed until 15:00 on Sunday 6 April. I drove with Konstanz mathematicians to Oberwolfach, arriving at 17:00. I reluctantly began my journey back to Urbana at 15:00 on Friday 11 April. (Times rounded to the nearest hour.)

While I was in Konstanz I gave the colloquium "Hilbert's 17th Problem during the Late 20th Century" at 17:00 on Thursday 3 April, and the Oberseminar "Ternary forms with lots of zeros" at 13:30 on Friday 4 April, in each case to an audience of approximately 20 faculty and graduate students. The Oberwolfach Workshop was "1415: Real Algebraic Geometry With A View Toward Systems Control and Free Positivity". I gave a morning survey talk "Some old and new results and examples on psd forms which are not sos" at 09:15 on Thursday 10 April, to the group. These three talks were all "blackboard" and not "beamer", so I cannot show you the .pdf's. I should be writing an extended abstract [7] for the Oberwolfach talk later this month, but your deadline comes first!

While in Konstanz. I had extensive and detailed conversations with Prof. Salma Kuhlmann and her Ph.D. student Ms. Charu Goel, regarding Goel's thesis on even symmetric forms which are not a sum of squares of forms. Her thesis relies heavily on an unpublished 1980 manuscript [4] written by M. D. Choi, T. Y. Lam and myself called "Real psd symmetric quartic forms". My own Ph.D. student William Harris drew on this manuscript for his 1992 thesis at the University of Illinois, and his 1999 paper [5], both of which have also been studied by Goel. I spent several hours per day in Konstanz talking with Prof. Kuhlmann and Ms. Goel about this work, and how the ideas in it might be applied to the thesis (I spent more time alone thinking about how to answer their questions and trying to remember what I was thinking 35 years ago!) In addition, I had a number of other more general conversations with other faculty at Konstanz, including Markus Schweighofer, Claus Scheiderer and Alex Prestel and with many of the graduate students working in the real algebra group.

In Oberwolfach, I attended all other talks (except for the ones on Friday afternoon after I left). I had additional conversations with the Konstanz group, with current and future collaborators Vicki Powers and Greg Blekherman and other colleagues,

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Date: April 15, 2014.

This research stay was partially supported by the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach.

especially Etienne de Klerk, Jean Lasserre, Monique Laurent, Murray Marshall and Konrad Schmüdgen.

I was also contacted by the organizers of the Workshop about making a contribution to the "Snapshots of Modern Mathematics from Oberwolfach" series, and agreed to do so, but have not started working on it yet.

In the absence of any papers yet produced as a result of this meeting, let me mention two topics as part of an ongoing collaboration with Greg Blekherman. More detailed information about this can be found in our research report [2] and in the forthcoming [7].

In 1980, Choi, Lam and I [3] proved that there is an integer $\alpha(k)$ with the property that if $p(x, y, z)$ is a real psd ternary form of degree $2k$ which has more than $\alpha(k)$ distinct zeros (viewed projectively), then there exists and indefinite ternary form h so that $p = h^2q$. We showed that $\alpha(2) = 1$, $\alpha(2) = 4$, $\alpha(3) = 10$, and $k^2 \le \alpha(k) \le \frac{3}{2}k(k-1)+1$ for $k \ge 4$. In the current work, we show that $\alpha(4) \ge 17$ and note from [5] that $\alpha(5) \ge 30$. Greg and I conjecture that $\alpha(k) \ge k^2 + 1$ for all $k \ge 3$, and using these examples, and results from [3] can prove it for all but five values of k, the largest being $k = 23$. In 1893, Hilbert [6] proved that for a psd form p of degree $2k$, there exists a psd form p_1 of degree $\le 2k-4$ so that pp_1 is a sum of three squares of forms. We discuss this multiplier for several of the examples with many zeros, and show uniqueness in minimal degree in two cases.

The second topic involves several well-known ternary sextics which are psd and not sos; known as the Motzkin form and the Robinson form. Part of our work will be to construct and discuss extremal hyperplanes which separate these forms from the cone of sos ternary sextics; this continues the work of my collaborator [1].

References

[1] G. Blekherman, *Nonnegative polynomials and sums of squares*, J. Amer. Math. Soc. **25** (2012), no. 3, 617635, MR2904568.

[2] G. Blekherman and B. Reznick, *PSD ternary forms with lots of zeros*, Research Report for MFO-1415.

[3] M. D. Choi, T. Y. Lam, B. Reznick, *Real zeros of positive semidefinite forms, I* , Math. Z. **171** (1980), 126. MR0566480 (81d:10012).

[4] M. D. Choi, T. Y. Lam, B. Reznick, *Psd symmetric quartic forms*, unpublished, c. 1980.

[5] W. Harris, *Real even symmetric ternary forms*, J. Algebra **222** (1999), 204245, MR1728161 (2001a:11070).

[6] D. Hilbert *Über ternäre definite Formen*, Acta Math. **17** (1893), 169197, MR1554835.

[7] B. Reznick, Extended abstract for Oberwolfach Workshop 1415, in preparation.

Department of Mathematics, University of Illinois at Urbana-Champaign, Urbana, IL 61801

E-mail address: reznick@math.uiuc.edu

Scientific Activity Report as Simons Visiting Professor

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

1 Scientific Activities at Universität des Saarlandes

I visited the Universität des Saarlandes as the Simons Visiting Professor 26 April to 6 May 2014. My primary mathematical contact during this visit was Professor Joerg Eschmeier, an expert in spectral theory and operator theory. During this visit we discussed two mathematical questions. While we weren't immediately able to resolve the questions, we were able to gain a better understanding of the issues related to them.

We first discussed the Taylor spectrum for Besov-Sobolev spaces on the unit ball. In a recent short unpublished note with Costea and Sawyer, I have computed the Taylor spectrum for a tuple of multiplication operators on the Besov-Sobolev spaces of the unit ball in several dimensions. Conversely, Eschmeier and Douglas have obtained a collection of conditions that allow them to deduce, given that the Taylor spectrum is known, the essential Taylor spectrum. Hence, coupling these two results, we are able to now compute the essential Taylor spectrum on the Besov-Sobolev space. This observation would be suitable for publication as a conference proceedings, and the next time such a paper is requested, these results will be formalized and written up for such a venue.

We next discussed a very interesting and challenging conjecture of Richter and Sundberg about the behavior of the spectrum of the tuple of multiplication operators by the coordinate functions on the Drury-Arveson Hardy space. In particular, Richter and Sundberg conjecture an explicit characterization of the spectrum on the boundary of the unit ball. Eschmeier has found a translation and connection of the conjecture into one about a Corona problem for the Drury-Arveson space; a topic I am very familiar with. While the current version of the Corona problem does not allow one to immediately resolve the conjecture, it does suggest a method to further analyze the conjecture which could ultimately lead to a resolution.

While visiting the University, I gave a Colloquium talk on "The Corona Problem." The Corona Problem is a fundamental question in function theory and operator theory that asks for necessary and sufficient conditions for a collection of functions to generate the Banach algebra of bounded analytic functions as an ideal. The original solution to this problem was obtained by Carleson in 1962, and has since served as an impetus for numerous other lines of inquiry and research. During the Colloquium I emphasized several different connections, in particular the connections the Corona problem has to complex geometry and operator theory, and the connections the Corona problem has to Besov-Sobolev spaces of analytic functions.

2 Scientific Activities at Mathematisches Forschungsinstitut Oberwolfach

I was a participant in the workshop "Hilbert Modules and Complex Geometry" at MFO during 21-25 April 2014. This workshop focused on the interactions and connections between operator theory, complex geometry, and function theory. As a participant in this workshop I gave a research talk that highlighted the role of Carleson measures of reproducing kernel Hilbert spaces of analytic functions. In particular, my talk, "Carleson Measures for Hilbert Spaces of Analytic Functions" discussed my recent work where a characterization of Carleson measures of spaces of analytic functions is obtained via the well-known T1 conditions in harmonic analysis.

During this workshop I also interacted with several participants in terms of potential future mathematics collaborations. With Kelly Bickel, we discussed matrix-weighted T1 theorems and how it should be possible to obtain a result in the style of Nazarov, Treil, and Volberg in terms of when certain well-localized operators are bounded on matrix weighted spaces. This result is currently being written up for publication. With Richard Rochberg, some discussions about computing the Dixmier trace of paraproducts took place. These discussions are still in the preliminary stages. Finally, with Stefan Richter we discussed when the Carleson measures for the Dirichlet space are the same as the Carleson measures for a certain endpoint analogue of the Dirichlet space are in fact the same. We were able to prove that this is always the case; while this is a useful observation, it is unclear what the immediate value and benefits a result of this type will yield.

May 11, 2014

Activity Report
Simons Visiting Professor Program at Oberwolfach

Participant: Stephen Kudla (University of Toronto)
Meeting: Modular Forms, 27 April - 3 May, 2014
Organizers: J. Bruinier, A. Ichino, T. Ikeda, Ö. Imamoglu
Code: 1418
Host University: Tech. Universität Darmstadt
Host: Jan Hendrik Bruinier
Dates: 22 - 26 April and 3 - 9 May, 2014

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach. It consisted of two periods in Darmstadt and a week in Oberwolfach, as indicated. All publications arising from this research will acknowledge this support.

Research Activities in Darmstadt:

The main efforts during this period were devoted to work an ongoing joint project with Jan Bruinier and Jens Funke (Durham). The overall problem is to construct explicit automorphic Green currents for special cycles of codimension $n > 1$ in the arithmetic quotients of the hermitian symmetric space attached to an orthogonal group of signature $(m, 2)$. Several such constructions are known in the case of divisors. One of them, due to Bruinier, involves taking a kind of archimedean local theta lifting of an M-Whittaker function attached to a principal series representation of $\mathrm{SL}_2(\mathbb{R})$. In contrast to the smooth and rapidly decreasing Whittaker functions which normally arise, these function grow rapidly at infinity, and, according to very general results of Goodman and Wallach, they can only be defined for functions in the induced representation which lie is certain Gevrey classes. The main goal of the current phase of the project is to construct analogous functions for the degenerate principal series representations $I(s)$ for $\mathrm{Sp}_n(\mathbb{R})$, the real symplectic group of rank n. We made significant progress on such a construction by means of Goodman-Wallach operators. Already for $\mathrm{SL}_2(\mathbb{R})$ we found a more explicit way of expressing the Goodman-Wallach transform, improving the discussion of this case in the original paper. In particular, our construction yields precisely the M-Whittaker functions used by Bruinier and it provides a very satisfying explanation of the relation between integral representations

$$U(a,b;z) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-zt}\, t^{a-1}\, (t+1)^{b-a-1}\, dt,$$

and

$$M(a,b;z) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(b-a)} 2^{1-b}\, e^{\frac{1}{2}z} \int_{-1}^{1} e^{-\frac{1}{2}zt}(1+t)^{a-1}\, (1-t)^{b-a-1}\, dt,$$

of the classical confluent hypergeometric functions. Work on the analogous construction for $\mathrm{Sp}_n(\mathbb{R})$ is well under way, and we hope to finish this portion of the project by the fall of this year.
In addition, while in Darmstadt I had interesting mathematical discussions with Stefan Ehlen, Anna von Pippich, and Shaul Zemel.

Research Activities in Oberwolfach:

I gave the opening lecture of the meeting, describing my recent results concerning product formulas for Borcherds forms converging in a neighborhood of any 1-dimensional boundary component. More details are contained in the extending abstract that I am providing for the Oberwolfach Reports. Such products already occur in special cases in the work of Borcherds and Gritsenko. My main result is that such product occur for all Borcherds forms and that they can be obtained by a variant of the Fourier expansion calculations in Borcherds second Inventiones paper involving regularized theta lifts. I had very fruitful discussions with Gritsenko about these products, the associated formulas for Fourier-Jacobi coefficients, and their applications.
In addition, while at Oberwolfach, I had stimulating mathematical discussions with Jens Funke, Wee Teck Gan, Siddarth Sankaran, and Tonghai Yang.
I am very grateful to the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach for supporting this very productive period of research.

Sincerely,

Stephen S. Kudla
Professor and Canada Research Chair
Department of Mathematics
University of Toronto
40 St. George St., BA 6290
Toronto, Ontario M5S 2E4, Canada

Scientific Activity Report on Simons Visiting Professorship (SVP)
Ragnar-Olaf Buchweitz

1. In November 2013 I was granted a Simons Visiting Professorship as a guest of Prof. Henning Krause at the Fakultät für Mathematik of the Universität Bielefeld, Germany, after/before the Oberwolfach Workshop

Interactions between Algebraic Geometry and Noncommutative Algebra
Dates: 18 May - 24 May 2014
Organizers:
Markus Reineke, Wuppertal, Germany
J. Toby Stafford, Manchester, UK
Catharina Stroppel, Bonn, Germany
Michel Van den Bergh, Diepenbeek, Belgium

2. I left my home town Toronto, Canada, for Bielefeld on Thursday, May 8, 2014, where I arrived the next day, and stayed in Bielefeld until Sunday, May 17, 2014 when I traveled from Bielefeld to Oberwolfach to participate in the above named workshop.

I am very happy to acknowledge that my research stay in Germany from May 9 to May 25, first at Bielefeld and then at Mathematisches Forschungsinstitut Oberwolfach (MFO) *was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.*

While not part of the Simons Visiting Professorship award I took advantage of being in Germany to stay afterwards as a guest of Prof. Hubert Flenner at the Fakultät für Mathematik of the Ruhr-Universität Bochum, Germany, for five weeks, until June 28, 2014, when I returned home to Toronto, Canada. During that time I also gave a colloquium talk at the University of Osnabrück on June 4, 2014.

3. As concerns my scientific activities during that Simons Visiting Professorship award, I gave two talks in Bielefeld,

- On Tuesday, May 13, 2014, I delivered a 90–minute talk in the research seminar on Representations of Algebras organized by Prof. Henning Krause and his research team at the Fakultät für Mathematik of the Universität Bielefeld.

  Title: *Higher Representation–Infinite Algebras from Geometric Tilting Objects*

  We reported on joint work with Lutz Hille, Universität Münster, Germany, on the recent notion of higher representation–infinite algebras. We show that a tilting object in a triangulated category of geometric dimension d, a notion proposed by Bondal, has an endomorphism ring that is higher representation-infinite if, and only if, it pulls back to a tilting object on the virtual affine canonical bundle over that category if, and only if, the endomorphism algebra has minimal global dimension, equal to d, and the tilting object is sheaf-like.

  The endomorphism ring of the pullback then yields the corresponding higher preprojective algebra. This proves, for example, that any full cyclic strongly exceptional sequence, a notion due to Hille–Perling that comprises the classical notion of a helix, gives rise to such pairs of d-representation-infinite algebras and their accompanying higher $(d+1)$-preprojective algebras, thereby providing plenty of examples.

  Intriguingly, such algebras also arise on non-Fano varieties, such as the second Hirzebruch surface or some non-isolated quotient singularities defined by abelian subgroups of special linear groups.

Modulo an outstanding conjecture on the graded coherence of higher preprojective algebras and results of Minamoto, it follows that representation–infinite algebras are precisely the ones arising as endomorphism rings of minimal global dimension of sheaf–like tilting objects in triangulated categories of geometric dimension d.

- On Thursday, May 13, 2014 I spoke in the departmental Colloquium of the Fakultät für Mathematik of the Universität Bielefeld.

  Title: *Matrix Factorizations over Elliptic Curves*

  Given a nonzero polynomial P in n variables, a matrix factorization of P consists of a pair of square matrices A, B of same size with entries from the polynomial ring such that $AB = P \cdot \mathbb{I}$, where $\mathbb{I}$ stands for the appropriate identity matrix. If the polynomial is homogeneous, one might further require that the entries of the matrices are homogeneous as well and represent morphisms between graded free modules over the polynomial ring.

  Such matrix factorizations play a crucial role in the so-called Landau-Ginzburg models of String Theory in Physics.

  A fundamental result by Orlov implies as a special case that equivalence classes of indecomposable such graded matrix factorizations for a cubic polynomial that defines an elliptic curve in the projective plane are in a natural, though still largely mysterious bijection with the isomorphism classes of indecomposable objects in the derived category of coherent sheaves on that elliptic curve. The structure of the latter is known since Atiyah's classification of such sheaves in 1957.

  After recalling the background just described, I presented results by my current student Sasha Pavlov who uses this machinery to determine all possibilities for the degrees of the entries and sizes of such matrix factorizations and indicated how this will enable us to find all such matrix factorizations eventually.

  The results relate in a profound way to classical work of Lenzing and co-authors on weighted projective lines.

  This was also the opening talk in a Colloquium in honour of Prof. Helmut Lenzing's 75th birthday that saw five additional talks the next day. See http://www.math.uni-bielefeld.de/birep/meetings/lenzing2014/ for a list of the additional speakers and their talks.

  I was very pleased that several colleagues from other German universities (Düsseldorf, Hannover, Duisburg–Essen, Dortmund) came to Bielefeld just to attend that talk.

Because of that colloquium in honour of Lenzing and the Oberwolfach workshop right afterwards there were several international researchers in Bielefeld that week and I profited greatly, for example, from discussions with Profs. Osamu Iyama (Nagoya, Japan), Atsushi Takahashi (Osaka, Japan), Idun Reiten and Steffen Oppermann (NTNU, Trondheim, Norway). I had as well opportunity to discuss our ongoing project with Prof. Lutz Hille (Münster, Germany) who attended the Lenzing Colloquium, and to discuss many items of mutual interest with my host, Prof. Henning Krause, and his doctoral students and postdoctoral fellows. One of the outcomes is that I have tentatively agreed to return to Bielefeld this coming November to participate in a workshop commemorating the upcoming 20th anniversary of Maurice Auslander's death who was responsible for so much of the progress in representation theory of algebras for the forty years prior to his untimely death in 1994.

4. After the week in Bielefeld, I went to attend the workshop at the MFO in Oberwolfach, as stated above. There, I was asked to give a talk on *Geometric Tilting Objects*, a condensed version, suitable for the advanced audience, of the first talk in Bielefeld described in some detail above. As always, participation in the workshop was a wonderful experience. Even after 40 years, I am still very excited each time I visit the MFO and I still enjoy every minute of it. The hours spent there in discussions with colleagues young and old are simply priceless and always worth the lack of sleep that inevitably ensues.

In closing, I wish to thank the Simons Foundation and the MFO for allowing me this opportunity to engage fully in mathematical research both at Bielefeld and at the MFO.

Ragnar-Olaf Buchweitz
Professor of Mathematics
Department of Computer and Mathematical Sciences
University of Toronto Scarborough
1265 Military Trail
Toronto, ON, Canada
M1C 1A4

Mathematisches Forschungsinstitut Oberwolfach
Simons Visiting Professorship Report

Thomas Nevins

Visiting Period: 19 May to 31 May, 2014

I. Visit to MFO: 19 to 24 May. I participated in the workshop at MFO "Interactions between Algebraic Geometry and Noncommutative Algebra." I gave a lecture at the workshop on my recent joint work with G. Bellamy (Glasgow), C. Dodd (Toronto), and K. McGerty (Oxford).

I had many fruitful interactions during the workshop. Among these were the following:

1. Bellamy, Toby Stafford, and I had some discussions on a joint project, begun at MSRI in 2013, that we intend to resume in the coming months.
2. McGerty, Ben Webster, and I worked to resolve a not-yet-completely-settled point that would allow a stronger cohomological application of my work with McGerty (described below) to the problem of "hyperkähler Kirwan surjectivity." We seem to have made substantial progress.
3. Bellamy, Dodd, McGerty, and I worked on calculating some interesting examples in symplectic algebraic geometry; see discussion below.
4. Conversations with, and talks by, I. Losev, H. Franzen, and M. Reineke at the MFO workshop, among others, may have inspired a new direction in an ongoing joint project with T. Bridgeland.

I further learned much from the talks of, and had interesting conversations with, Jason Bell, Birge Huisgen-Zimmerman, Susan Sierra, Chelsea Walton, and many others.

I would like to mention that I think the organizers and MFO deserve special congratulations for this workshop: for me, at least, it was one of the best and most productive that I have been to anywhere in the last five years.

II. Visit to the University of Oxford: 24 to 31 May. I visited the University of Oxford for the week of 24 to 31 May. During that period, I gave a seminar talk in the Mathematical Institute at Oxford. I planned work on two main research directions for that week:

1. With Kevin McGerty of Oxford, I have been engaged in a long-term project, developing new tools for the study of $\mathcal{D}$-modules on algebraic varieties with group action and applying them. We completed two papers in winter 2013–2014: one which proves a new vanishing theorem for $\mathcal{D}$-modules, and another that establishes a general gluing procedure, or *recollement*, for the full category of $\mathcal{D}$-modules on a variety with group action. During my week at Oxford, we made significant progress in working out concrete consequences of these general results for an important example in geometric

representation theory, namely, the moduli stack of G-bundles on an algebraic curve C. The results we deduce have concrete consequences in the geometric Langlands program. Based on our progress that week, I have been steadily writing up these new results in the week since my return home.

As this work, and all the work carried out during these weeks, was partially supported by the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach, the support of those entities will be acknowledged in all resulting publications.

2. With McGerty, Gwyn Bellamy, and Chris Dodd, I have been working to establish a different kind of "categorical Morse theory" for quantizations of symplectic algebraic varieties. Our first paper on the subject was posted to the arXiv late in 2013; but it turned out to have an error. Fixing the error did not require changes to any of the main results of the paper, but it did lead to our discovering that there is a richer geometry for certain symplectic algebraic varieties (those with an action of $\mathbb{C}^*$ for which all downward orbits converge) than we had before understood. The four of us spent significant time at MFO trying to understand some basic examples of this richer geometry.

   Bellamy then also visited Oxford for 29-30 May. He, McGerty, and I spent those days resolving some technical issues for our next paper, which takes a further step toward categorical Morse theory for what we are calling "bionic symplectic varieties." Hyperkähler rotations of many such varieties appear prominently in *real* symplectic geometry as *Weinstein manifolds,* and we are working out tools that we hope will eventually allow us to prove that the Fukaya categories of such manifolds are realized by modules over deformation quantizations of the rotated algebraic varieties, constituting progress toward resolving a significant problem in the study of Fukaya categories that is largely open.

I am grateful to the MFO and the Simons Foundation for their support of my workshop participation and visit to Oxford. This was a very fruitful research period for me!

Nice, 22th june 2014

I have been invited by the organizers of the international conference: "Stochastic Analysis: around the KPZ universality class" to attend the meeting, which held in the Mathematisches Forschungsinstitut Oberwolfach from 1st to 7th june 2014. During the meeting I was also invited by Professor Martin Hairer, to give a talk with title

Derivation of the KPZ equation from particle systems,

which is a joint work with Milton Jara from IMPA, Brazil and Sunder Sethuraman from the University of Arizona, USA. The pdf of the talk is attached to this letter.

During the week of the meeting I have been working with Milton Jara on our research project related to the equilibrium fluctuations for the exclusion process with long jumps, which are given by a probability measure with heavy tails. The fluctuations for this model are described by the fractional KPZ equation. The proof uses a multi-scale argument which is done in two steps and a relatively compactness argument. This work is on its final form and will be submitted for publication in a high standard journal.

The two weeks after the meeting, I have been visiting Cédric Bernardin at the University of Nice. During those two weeks, we have been working on two problems that I describe below.

In the first problem we considered a stochastic model, which consists on a deterministic hamiltonian dynamics that is perturbed by an exchange noise, acting on all the cells of the system, and also a flip noise, acting just at the origin cell. The deterministic dynamics conserves several thermodynamical quantities of the system, but the perturbed system conserves only two thermodynamical quantities: the volume and the energy. Our interest is to prove the hydrodynamic limit for this model. Depending on the strength of the flip noise, the hydrodynamic equation for the volume is the transport equation with periodic boundary conditions, Robin's boundary conditions and Dirichlet's boundary conditions. The hydrodynamic equation for the energy is given in terms of the volume, but does not depend on the strength of the flip noise. For this model we also studied the non-equilibrium fluctuations for the energy and for the volume. The fluctuations are given by Gaussian processes with a covariance which depends on the strength of the flip noise. The estimates at the level of the space-time correlations are complicated and are overcome by the use of Fourier analysis. This is a joint work with Cédric

Bernardin and Adriana Neumann from the Federal University of Rio Grande do Sul.

In the second problem we considered the exclusion process with long jumps given by a probability measure with heavy tails. Depending on the probability function being symmetric or asymmetric we obtain upper and lower bounds for several additive functionals of the dynamics. For some functionals, the bounds are sharp in the sense that we can capture the correct scaling in order to have a non-trivial limit. In some cases we can also identify the limit which goes from the Brownian motion to the fractional Brownian motion. The asymmetric case is more demanding, but we could also obtain upper and lower bounds for the variance and in some cases, identify the limit process. This is a joint work with Cédric Bernardin and Sunder Sethuraman from the University of Arizona.

Finally, I would deeply like to thank the Simons Foundation and the Mathematisches Forschungsinstitut Oberwolfach for the financial support. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Patrícia Gonçalves
Assistant Professor
Pontifical Catholic University
Rio de Janeiro
Brazil

Scientific report on a visit to Prof Herbert Spohn (TUM, Munich)

May 14 - May 29, 2014

Statistical properties of directed polymers were subject of intensive studies in the past decade. From the mathematical standpoint directed polymers are random walks in random environment. Consider a discrete time random walks on the lattice $\mathbb{Z}^d$. We shall assume that time t belongs to $\mathbb{Z}^1$. If the position of the random walk at time $t = i$ is given by $S(i) \in \mathbb{Z}^d$, then next moment of time it either jumps at one of the nearest neighbours, or stays at the the same site: $\|S(i+1) - S(i)\| \leq 1$. Condition above corresponds to the so-called "lazy" random walks. The random environment is given by a random potential. We shall assume that every point (x, i) of the space-time $\mathbb{Z}^{d+1}$ is equipped with a random variable $V^\omega(x, i)$, where by ω we denote the whole configuration of the random field $\{V(x, i), (x.i) \in \mathbb{Z}^{d+1}\}$. For a random walk path $S(i), 0 \leq i \leq n$ of length n define its action

$$A_n^\omega(S) = \sum_{i=0}^{n} V^\omega(S(i), i).$$

Notice that A_n is a random variable which depends on a realization ω of the random field $\{V(x, i), (x.i) \in \mathbb{Z}^{d+1}\}$. Now we can define a polymer measure P_n^ω on the set of all paths starting from a fixed point:

$$P_n^\omega(S) = \frac{1}{Z_{n,\beta}^\omega} \exp\left(-\beta A_n^\omega(S)\right), \tag{1}$$

where $\beta \geq 0$ is the inverse temperature, and the normalization factor $Z_{n,\beta}^\omega$ called a partition function is given by:

$$Z_{n,\beta}^\omega = \sum_S \exp\left(-\beta A_n^\omega(S)\right).$$

Note that we consider so-called quenched setting where the configuration ω is assumed to be fixed. Zero-temperature polymers correspond to a limit $\beta \to \infty$. In this case the polymer measure is supported on paths which minimize the action A_n^ω. If the probability distribution of the field $\{V(x, i), (x.i) \in \mathbb{Z}^{d+1}\}$ is continuous then for a fixed starting point such a path is unique with probability 1. In what follows such action-minimizing paths are called optimal paths and denoted $\bar{S}$. The main goal is to understand their asymptotic statistical and scaling properties in the limit $n \to \infty$. A related problem is a study of asymptotic statistical properties of the minimal action value $\bar{A}_n^\omega = A_n^\omega(\bar{S})$. The most interesting situation which was recently intensively studied corresponds to $d = 1$. The case of independent identically distributed (iid) random variables $\{V^\omega(x, i), (x, i) \in \mathbb{Z}^{1+1}\}$ belong to the so-called KPZ (Kardar - Parizi - Zhang) universality class. In this case there are very precise conjectures about asymptotic properties of the polymer measure and optimal paths. In few cases these conjectures has been proved rigorously. Notice that it is not necessary to specify the probability distribution for disorder. It is believed that the asymptotic behaviour does not depend on it provided that a random variable V has certain number of finite moments. In other words the asymptotic behaviour is universal.

Different models correspond to different types of random potential $\{V^\omega(x,i)\}$. One of them was already discussed above. This is the KPZ model which corresponds to independent identically distributed random variables $\{V^\omega(x,i),(x,i)\in\mathbb{Z}^{1+1}\}$. Recently we suggested to study the problem of optimal paths for other statistical models for the field $\{V^\omega(x,i),(x,i)\in\mathbb{Z}^{1+1}\}$. These models correspond to a product-type random environment. Namely, consider a random field $\{F(x),x\in\mathbb{Z}^1\}$ of iid random variables, and another field $\{b(i),i\in\mathbb{Z}^1\}$ which is also assumed to be iid. We also assume that the fields $\{F(x),x\in\mathbb{Z}^1\}$ and $\{b(i),i\in\mathbb{Z}^1\}$ are statistically independent. Now, let us define $V(x,i)=F(x)b(i),(x,i)\in\mathbb{Z}^2$. Roughly speaking such potentials correspond to the spatially disordered setting. In addition a polymer is embedded in an external potential fluctuating in time. We shall assume that the probability distribution for V has compact support, and it is given by the density $p(V)$ which vanishes outside of the closed interval $[-C,C]$ and positive everywhere inside $[-C,C]$. In addition we assume that random variables b have zero expectation: $\mathbb{E}(b)=0$. The asymptotic behaviour for this model is different from the KPZ model, but it is also universal. In a joint paper with Yuri Bakhtin and Jeremy Voltz we show that critical exponents of the optimal action and of the polymer endpoint are both equal to 2/3. And after normalization on $n^{2/3}$ both the probability distributions for the centered optimal action $A_n^\omega(\bar{S}_n)+Cn$ and the polymer end-point $\bar{S}_n(n)$ converge to universal limit laws which can be described explicitly. One can also show that the shape function corresponding to the optimal action of the point-to-point optimal paths has a corner at the origin. This explains why both critical exponents in this model coincide. While one can show that the shape function is non-linear, we conjecture that it has a linear piece near the origin.

The mathematical analysis of this model is possible since one can describe the preferred positions for an optimal path. During the visit to Munich we discussed another model of a product-type. Instead of one field $\{F(x),x\in\mathbb{Z}^1\}$ we consider two statistically independent copies of it, $\{F_1(x),x\in\mathbb{Z}^1\}$ and $\{F_2(x),x\in\mathbb{Z}^1\}$. We also consider two independent copies $\{b_1(i),i\in\mathbb{Z}^1\}$ and $\{b_2(i),i\in\mathbb{Z}^1\}$ of the process $\{b(i),i\in\mathbb{Z}^1\}$. We then define $V(x,i)=F_1(x)b_1(i)+F_2(x)b_2(i),(x,i)\in\mathbb{Z}^2$. This model is substantially different from the previous one. At present there are no mathematically rigorous results for it. Preliminary numerical studies indicate that both critical exponents which we discussed above are universal and take values close to 5/6. I have discussed possible mathematical approaches with Herbrt Spohn and Michael Prahoffer. We also discussed with Michael Prahoffer how one can possibly prove that the shape function has a transition from linear to nonlinear behaviour. A simplified model was proposed where the random field $\{F(x),x\in\mathbb{Z}^1\}$ is a Bernoulli sequence taking values ±1. Although it is likely that a shape function for this model does not have an above transition, the analysis provide additional information about the shape function for an original setting.

Overall it was an interesting visit which stimulated an exchange of ideas in a very active an important area of statistical mechanics. I am very grateful to Simmons foundation for supporting my visit to Munich.

Konstantin Khanin
June 20, 2014

Scientific Activity Report*

Davar Khoshnevisan

July 2, 2014

This visit and fellowship served two purposes: On one hand, it allowed the PI to visit the Oberwolfach Institute for the workshop, "Stochastic Analysis: Around the KPZ Universality Class" [Workshop ID 1423] that was held at the Oberwolfach Institute from June 1, 2014 to June 7, 2014. The general area of the KPZ universality class is an exciting new area of probability theory, based on older problems that arose earlier in statistical mechanics. The meeting was attended by a large number of leading experts on topics that are related directly to the subject of the workshop. Those topics include theoretical physics, statistical mechanics, stochastic analysis and stochastic PDEs, interacting particle systems, random matrices, integrable systems, ...

The PI talked about "multifractality and intermittency," based on his recent collaborative work with K. Kim and Y. Xiao. An abstract of the talk has been reported in Oberwolfach Reports. Most importantly, the workshop allowed for free interactions, many extremely useful, among the participants. This was a quite diverse group of mathematicians and physicists, and a good number of ideas were exchanged, many of which are likely to have impact on the field in the future.

The second half half of the PIs visit was to visit the Stochastics group at

*This research stay was partially supported by the *Simons Foundation* and by the *Mathematische Forschungsinstitut Oberwolfach.*

TU-Dresden [June 7, 2014 through June 14, 2014]. The host was Professor Rene Schilling, the head of the group, who is a collaborator of the PI. The PI gave a seminar talk at Dresden and worked with the group in some informal and a few more formal capacities. Several meetings were held wherein the members of the Dresden group [particularly impressive the younger members] described their recent works and its potential relations to the PIs talk.

In summary, this was an exciting visit; the PI had the opportunity to work with several top researchers in Oberwolfach and Dresden. It is the PI's hope that some of these exchanges might lead to future mathematical innovations. The PI is grateful to the Oberwolfach Institute and the Simons Foundation for making this exciting mathematical visit possible. [Without doubt, this particular combination of scientific events would otherwise not have been accessible to the PI.]

Scientific Activity Report

Anton Petrunin

Dates.

- Oberwolfach Workshop: Geometrie, 15–21 June 2014.
- University of Cologne: 3–14 and 22–30 June 2014.

Talks and Lectures.

- *Smoothing and Facetting,* FMO, June 19, 2014;
- Minicourse: *Piecewise distance preserving maps with applications and amusements,* Univeristy of Cologne, June 23–24, 2014;
- *Smoothing and Facetting,* Oberseminar Geometrie, Topologie und Analysis, Univeristy of Cologne, June 27, 2014;

Research collaboration. During my stay at University of Cologne, together with Alexander Lytchak and Vitali Kapovitch, we were working on integral geometry of Alexandrov spaces; in particular on generalization of Liouville's theorem on invariance of the natural measure on the tangent bundle with respect to geodesic flow.

Such a generalization of Liouville's theorem is interesting on its own, but it also have applications. Let me mention that Liouville's theorem would imply existence of infinte geodesics in almost any direction of Alexandrov's space without boundary — this is a long standing open problem. In particular, passing to the limit, we could then obtain a new proof of existence of quasigeodesic in any direction. The existence of quasigeodesic is known but the available proof is quite involved and it would be interesting to find a new proof.

We were able to get partial results which hopefully will lead to the solution. In particular, we show that Liouville's theorem can be reduced to the following formula in integral geometry of Alexandrov spaces; see the conjecture below.

Boundary measure. Assume M is a compact Riemannian m-dimensional manifold with possibly nonempty boundary ∂M. Let us denote by $v_\varepsilon(x)$ the volume of ε-ball centered at point x in M. It is well known (and easy to prove) that

$$\int\limits_M v_\varepsilon\, d\mathrm{vol} = \alpha_m \cdot \mathrm{vol}_m M \cdot \varepsilon^m + \beta_m \cdot \mathrm{vol}_{m-1}\partial M \cdot \varepsilon^{m+1} + o(\varepsilon^{m+1}), \qquad (*)$$

where α_m and β_m are constants which depend only on the dimension m.

Conjecture. *The identity* $(*)$ *holds for any Alexandrov space.*

In particular, it would imply that if Alexandrov space A has no boundary then

$$\int_A v_\varepsilon \, d\mathrm{vol} = \alpha_m \cdot \mathrm{vol}_m A \cdot \varepsilon^m + o(\varepsilon^{m+1}).$$

We can show that the last statement alone implies Liouville's theorem.

We were also able to show that for any compact Alexandrov space A there is a locally bounded measure δ such that

$$\int_A v_\varepsilon \, d\mathrm{vol} \le \alpha_m \cdot \mathrm{vol}_m A \cdot \varepsilon^m + \delta(A) \cdot \varepsilon^{m+1} + o(\varepsilon^{m+1}).$$

The measure δ should be considered as a measure-theoretic analog of boundary, it is defined on a wide variety of metric spaces and we believe it deserves further study.

In some cases we can show that δ vanish. For example it is true for Alexandrov spaces which admit approximation by Riemannian manifolds of the same dimension. In particular it holds for two-dimensional Alexandrov spaces without boundary. (The paper should be available on arXiv soon.)

In addition to the main topic described above we had a number of discussion in geometry of Alexandrov spaces with curvature, party connected to the book on Alexandrov geometry which we (S. Alexander, V. Kapovitch and I) are currently writing.

The acknowledgment. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Scientific activity report

Igor Dolgachev

July 14, 2014

This is to report on my scientific activity as a participant of the Oberfolfach workshop on Classical Algebraic Geometry during the week June 30-July 4 and the subsequent visit, as a Simons Visiting Professor, of the Technical Institute at Münich during the week July 5-July 12.

As a participant of the workshop on Classical Algebraic Geometry, I did what I was expected to do; attending the lectures and discussing mathematics with other participants. Besides, in the spare time I was working with Professor Alessandro Verra from Rome on our outgoing project on the problem of unirationality of the moduli spaces of polarized complex Enriques surfaces (more about this later). I was also fortunate to meet in Oberfolfach Professor Viacheslav Kharlamov from Strasburg who was staying in Oberfolfach for a longer time. We were working on the problem of real structures on algebraic surfaces. It is not yet known whether there exists a nonsingular projective surface defined over reals that admits infinitely many non-isomorphic real structures. We know that such a surface X must be a rational surface that admits a group G of automorphisms with infinitely many conjugacy classes of real involutions. The group G, via it action on the cohomology $H^2(X,\mathbb{Z})$ can be identified (up to finite groups) with a subgroup of the orthogonal group of a certain hyperbolic lattice. Hence it can be considered as a discrete group of isometries of a hyperbolic space $\mathbb{H}^n$. If the group is geometrically finite, then it is known that it contains only a finite set of conjugacy classes of involutions. So, we have to find an example a surface defined over reals such that it contains a non-geometrically finite group of automorphisms. A geometrically finite discrete group of isometries of a hyperbolic space is finitely generated (but the converse is not true in dimension > 2). In fact, we do not even know that there exists an example of a surface with not finitely generated group of automorphisms. So, we spent most of our discussions with trying to construct an example of a rational surface with non-finitely generated group of automorphisms. It seems it is a very hard problem, and we will certainly pursue it in our future work on the problem.

My host in Munich was Professor Christian Liedtke. Last year he kindly agreed to cooperate with me on writing the second part of a monograph on Enriques surfaces. The first part, joint with Francois Cossec, was published in 1989 by Birkhäuser. The main feature of our former and current project is to treat Enriques surfaces over an algebraically closed fields of arbitrary characteristic p. In the case $p = 2$, the geometry of Enriques surfaces is drastically different from other cases of p, as was first observed in the earlier foundational work of D. Mumford and E. Bombieri on classification of algebraic surfaces over fields of positive characteristic. Christian is a young mathematician who has already distinguished himself by proving two important and old conjectures in the theory of Enriques surfaces and K3 surfaces over fields of positive characteristic. In particular, he has solved the problem of liftability of Enriques surfaces to characteristic 0 and described completely the moduli stack of Enriques surfaces in characteristic 2. The latter consists of two irreducible components formed by classical and μ_2-surfaces that intersect along the locus of α_2-surfaces. In his other outstanding work, he has proved the unirationality of supersingular K3 surfaces in any characteristic (the case $p = 2$ it was known and proved by Alexei Rudakov and Igor Shafarevich). The topic of the second part of the book includes the theory of moduli of Enriques surfaces, and here his expertise will be essential for our project. Fortunately, the week I spent in Munich, Christian was mostly free from other duties, and we were able to work during long hours most of the week. As a byproduct of our discussion about the book project, we were able to prove a very nice new result about the unirationality of the moduli space of polarized Enriques surface. It is known that all such spaces are rationally dominated by the moduli space of Enriques surfaces together with a basis in its lattice of divisor classes modulo numerical equivalence, we call it the master space. So to prove that all moduli spaces of polarized surfaces are unirational it suffices to prove the unirationality of the master space. As I mentioned before, our attempt with Verra to prove this in the case of characteristic 0 is still unsuccessful. However, quite surprisingly, the case of the characteristic 2 turned out to be somewhat easier, and we have succeeded in proving that the master space of classical Enriques surfaces (characterized by the condition that the canonical class is not zero, as in the case of characteristic 0) is unirational (modulo of the Ogus-Torelli theorem for supersingular K3 surfaces which is known in characteristic $p > 2$ but is believable that it is also true in characteristic $p = 2$).

In conclusion, I would like to acknowledge the support of the Simons Foundation that allowed me to work very productively during my two weeks stay in Germany.

Respectfully submitted by

Igor Dolgachev

Igor Dolgachev

Professor of Mathematics, Emeritus

University of Michigan

Simons Visiting Professorship: Scientific Activity Report
Henri Darmon

Dear Prof. Huisken,

Please find below the Scientific Activity Report associated to my one-month stay in Germany as a Simons Visiting Professor (SVP) in July 2014. I apologise for being late in sending it in: I had not realised that the report needed to be sent in within two weeks of the end of the visit.

My visit to Germany was divided into three periods:

One week visit in Heidelberg *(June 30-July 4).* During this week I had scientific exchanges with my Heidelberg colleagues, most notably Otmar Venjakob and Gebhard Boeckle. On Thursday I gave the Emil Artin Vorlesung, on the topic of "Elliptic curves and explicit class field theory", a colloquium-style lecture aimed at a fairly general audience. (Normally this lecture is given on late Friday afternoons, but it was rescheduled in order to not interfere with the World Cup game between France and Germany). I also gave a seminar lecture the next morning on a related topic, but at a more specialised level, entitled *The Birch and Swinnerton-Dyer conjecture for ring class fields of real quadratic fields.*

One week at Oberwolfach *(July 7-11).* I participated in the workshop *Algebraishe Zahlentheorie* organised by Ben Howard, Guido Kings, Ramdorai Sujatha, and Otmar Venjakob, and gave the opening lecture, entitled "Euler systems and the Birch and Swinnerton-Dyer conjecture". The contents of this lecture are summarised in my Oberwolfach report, which I attach to this activity report. The week in Oberwolfach also gave me the opportunity to work with two of my close scientific collaborators who were also present at the meeting, Massimo Bertolini from Essen and Victor Rotger from Barcelona; during this time we completed the write-up of an article on p-adic families of Beilinson Flach elements and the arithmetic of elliptic curves.

Two weeks at Universität Essen *(July 14-25).* During this period I visited the University of Essen where I interacted with Massimo Bertolini. During these two weeks we had fruitful exchanges on the arithmetic of elliptic curves. We were notably able to complete our work with Rotger on p-adic families of Beilinson Flach elements. This work, entitled "Beilinson-Flach elements and Euler systems II: the Birch and Swinnerton-Dyer conjecture for Hasse-Weil-Artin L-series", is now publicly available at `http://www-ma2.upc.edu/vrotger/publicacions_en.html`. We also discussed further projects growing out of this collaboration, notably a general strategy for relating the "generalised Kato classes" studied in our earlier works to Heegner points, thereby proving a conjecture of Perrin-Riou. In the course of these two weeks, I also gave a lecture in the number theory seminar in Essen, with the same title as my Oberwolfach lecture, and on a closely related theme. One of my former students and one of my former postdocs (Marc Masdeu and Xavier

Guitart) also happened to be visiting Essen at that time and I enjoyed several scientific exchanges with them concerning their ongoing projects aiming to perform efficient calculations of Stark-Heegner points.

Overall my month in Germany was extremely enjoyable and productive and I want to thank the MFO as well as the Simons Foundation for its support. If there is anything else that you would like to see in this report, please do not hesitate to contact me again.

With my best regards,

Henri Darmon

Attachments:

-- Poster for Emil Artin Vorlesung

-- Oberwolfach Report

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

FAKULTÄT FÜR MATHEMATIK
UND INFORMATIK

UNIVERSITÄT
HEIDELBERG
ZUKUNFT
SEIT 1386

3. EMIL ARTIN VORLESUNG
ELLIPTIC CURVES AND EXPLICIT CLASS FIELD THEORY

Prof. Dr. Henri Darmon
(McGill University, Canada)

3. Juli 2014
Mathematisches Institut
Hörsaal 2
Im Neuenheimer Feld 288
16.45 Uhr Kaffee
17.15 Uhr Vortrag

Emil Artin
(*03.03.1898 †20. 12.1962)

www.match.uni-heidelberg.de

Euler systems and the Birch and Swinnerton-Dyer conjecture

Henri Darmon
(joint work with Massimo Bertolini, Victor Rotger)

The Birch and Swinnerton-Dyer conjecture for an elliptic curve $E/\mathbb{Q}$ asserts that

$$\operatorname{ord}_{s=1} L(E,s) = \operatorname{rank}(E(\mathbb{Q})), \tag{1}$$

where $L(E,s)$ is the Hasse-Weil L-function attached to E. The scope of the conjecture can be broadened somewhat by introducing an Artin representation

$$\varrho : G_{\mathbb{Q}} \longrightarrow \operatorname{Aut}(V_\varrho) \simeq \mathbf{GL}_n(\mathbb{C}), \tag{2}$$

and studying the Hasse-Weil-Artin L-function $L(E,\varrho,s)$, namely, the L-function attached to $H^1_{\mathrm{et}}(E_{\bar{\mathbb{Q}}},\mathbb{Q}_p)\otimes V_\varrho$, viewed as a (compatible system of) p-adic representations. The "equivariant Birch and Swinnerton-Dyer conjecture" states that

$$\operatorname{ord}_{s=1} L(E,\varrho,s) = \dim_{\mathbb{C}} \hom_{G_{\mathbb{Q}}}(V_\varrho, E(H)\otimes\mathbb{C}), \tag{3}$$

where H is a finite extension of $\mathbb{Q}$ through which ϱ factors. Denote by $\mathrm{BSD}_r(E,\varrho)$ the assertion that the right-hand side of (3) is equal to r when the same is true of the left-hand side. Virtually nothing is known about $\mathrm{BSD}_r(E,\varrho)$ when $r>1$. For $r\le 1$, there are the following somewhat fragmentary results, listed in roughly chronological order:

Theorem (Gross-Zagier 1984, Kolyvagin 1989) *If ϱ is induced from a ring class character of an imaginary quadratic field, and $r\le 1$, then $BSD_r(E,\varrho)$ holds.*

Theorem A (Kato, 1990) *If ϱ is abelian (i.e., corresponds to a Dirichlet character), then $BSD_0(E,\varrho)$ holds.*

Theorem B (Bertolini-Darmon-Rotger, 2011) *If ϱ is an odd, irreducible, two-dimensional representation whose conductor is relatively prime to the conductor of E, then $BSD_0(E,\varrho)$ holds.*

Theorem C (Darmon-Rotger, 2012) *If $\varrho=\varrho_1\otimes\varrho_2$, where ϱ_1 and ϱ_2 are odd, irreducible, two-dimensional representations of $G_{\mathbb{Q}}$ satisfying:*

1. $\det(\varrho_1)=\det(\varrho_2)^{-1}$, *so that ϱ is isomorphic to its contragredient representation;*
2. *ϱ is regular, i.e., there is a $\sigma\in G_{\mathbb{Q}}$ for which $\varrho(\sigma)$ has distinct eigenvalues;*
3. *the conductor of ϱ is prime to that of E;*

then $BSD_0(E,\varrho)$ holds.

This lecture endeavoured to explain the proofs of Theorems A, B, and C, emphasising the fundamental unity of ideas underlying all three.

The key ingredients are certain global cohomology classes

$$\kappa(f,g,h)\in H^1(\mathbb{Q},V_f\otimes V_g\otimes V_h(c))$$

attached to triples (f,g,h) of modular forms of respective weights (k,ℓ,m); here V_f, V_h and V_g denote the Serre-Deligne representations attached to f, g and h, and it is assumed that the triple tensor product of Galois representations admits a Kummer-self-dual Tate twist, denoted $V_f \otimes V_g \otimes V_h(c)$. (This is true when the product of nebentype characters associated to f, g and h is trivial.)

When f, g and h are all of weight two and level dividing N, and f is cuspidal, associated to an elliptic curve E, say, the class $\kappa(f,g,h)$ admits a geometric construction via p-adic étale regulators/Abel-Jacobi images of

(1) Beilinson-Kato elements in the higher Chow group $\mathrm{CH}^2(X_1(N),2)$ of the modular curve $X_1(N)$, when g and h are Eisenstein series of weight two arising as logarithmic derivatives of suitable Siegel units;
(2) Beilinson-Flach elements in the higher Chow group $\mathrm{CH}^2(X_1(N)^2,1)$ when g is cuspidal and h is an Eisenstein series;
(3) Gross-Kudla-Schoen diagonal cycles in the Chow group $\mathrm{CH}^2(X_1(N)^3)$, when all forms are cuspidal.

When g and h are of weight one rather than two, and hence, are associated to certain (possibly reducible) odd two-dimensional Artin representations, the construction of $\kappa(f,g,h)$ via K-theory and algebraic cycles ceases to be available. The class $\kappa(f,g,h)$ is obtained instead by a process of p-adic analytic continuation, interpolating the geometric constructions at all classical weight two points of Hida families passing through g and h in weight one, and then specialising to this weight. The resulting $\kappa(f,g,h)$ is called the *generalised Kato class* atttached to the triple (f,g,h) of modular forms of weights $(2,1,1)$.

The generalised Kato classes arising from (p-adic limits of) Beilinson-Kato elements, Beilinson-Flach elements, and Gross-Kudla-Schoen cycles are germane to the proofs of Theorems A, B and C respectively. The key point in all three proofs is an *explicit reciprocity law* which asserts that the global class $\kappa(f,g,h)$ is *non-cristalline at* p pecisely when the classical central critical value $L(f\otimes g\otimes h,1) = L(E,\varrho,1)$ is non-zero. The non-cristalline classes attached to (f,g,h) (of which there are actually four, attached to various choices of ordinary p-stabilisations of g and h) can then be used (by a standard argument involving local and global Tate duality) to conclude that the natural inclusion of $E(H)$ into $E(H\otimes\mathbb{Q}_p)$ becomes zero when restricted to $\varrho_g\otimes\varrho_h$-isotypic components, and hence, that $\hom_{G_{\mathbb{Q}}}(V_\varrho, E(H)\otimes\mathbb{C})$ is trivial when $L(E,\varrho,1)\neq 0$.

The lecture strived to set the stage for the two that immediately followed, which were both devoted to further developments arising from these ideas:

(1) Victor Rotger's lecture studied the generalised Kato classes $\kappa(f,g,h)$ when $L(f,g,h,1)=0$. In that case, they belong to the Selmer group of E/H, and can be viewed as p-adic avatars of $L''(E,\varrho,1)$;
(2) Sarah Zerbes' lecture reported on [LLZ1], [LLZ2], [KLZ] in which the study of Beilinson-Flach elements undertaken in [BDR] is generalised, extended and refined. By making more systematic use of the Euler system properties of Beilinson-Flach elements, notably the possibility of "tame deformations" at primes $\ell\neq p$, the article [KLZ] is also able to establish strong finiteness results for the relevant ϱ-isotypic parts of the Shafarevich-Tate group of E over H, in the setting of Theorem B.

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SCIENTIFIC REPORT OF FRANCESCO MAGGI, SVP FELLOW HOSTED AT THE UNIVERSITY OF ZÜRICH, JULY 2014

Since the pioneering work of Reifenberg there has been an ongoing interest into formulations of Plateau's problem involving the minimization of the Hausdorff measure on closed sets coupled with some notion of "spanning a given boundary". More precisely consider any closed set $H \subset \mathbb{R}^{n+1}$ and assume to have a class $\mathcal{P}(H)$ of relatively closed subsets K of $\mathbb{R}^{n+1} \setminus H$, which encodes a particular notion of "K bounds H". Correspondingly there is a formulation of Plateau's problem, namely the minimum for such problem is

$$m_0 := \inf\{\mathcal{H}^n(K) : K \in \mathcal{P}(H)\}\,, \tag{0.1}$$

and a minimizing sequence $\{K_j\} \subset \mathcal{P}(H)$ is characterized by the property $\mathcal{H}^n(K_j) \to m_0$. There are substantial difficulties related to the minimization of Hausdorff measures on classes of closed (or even compact) sets. Depending on the convergence adopted, these are either related to lack of lower semicontinuity or to compactness issues. My goal during the research staying at the University of Zürich has been to show that in some interesting cases these difficulties can be avoided by exploiting Preiss' rectifiability theorem for Radon measures [Pre87, DL08] in combination with the sharp isoperimetric inequality on the sphere and with standard variational arguments, noticeably elementary comparisons with spheres and cones. The key idea sparkled from a discussion with Camillo De Lellis and during my staying in Zürich we expanded upon this idea together with his postdoc Francesco Ghiraldin. A precise formulation of our main result, to appear shortly in a joint paper, is the following:

Definition 1 (Cone and cup competitors)**.** *Let $H \subset \mathbb{R}^{n+1}$ be closed. Given $K \subset \mathbb{R}^{n+1} \setminus H$ and $B_{x,r} = \{y \in \mathbb{R}^n : |x-y| < r\} \subset \mathbb{R}^{n+1} \setminus H$, the cone competitor for K in $B_{x,r}$ is the set*

$$\big(K \setminus B_{x,r}\big) \cup \big\{\lambda x + (1-\lambda) z : z \in K \cap \partial B_{x,r}\,, \lambda \in [0,1]\big\}\,; \tag{0.2}$$

a cup competitor for K in $B_{x,r}$ is any set of the form

$$\big(K \setminus B_{x,r}\big) \cup \big(\partial B_{x,r} \setminus A\big)\,, \tag{0.3}$$

where A is a connected component of $\partial B_{x,r} \setminus K$.

A family $\mathcal{P}(H)$ of relatively closed subsets $K \subset \mathbb{R}^{n+1} \setminus H$ is a good class if, for every $K \in \mathcal{P}(H)$, for every $x \in K$, for a.e. $r \in (0, \mathrm{dist}(x,H))$, and whenever L is the cone competitor or any cup competitor for K in $B_{x,r}$, one has

$$\inf\big\{\mathcal{H}^n(J) : J \in \mathcal{P}(H)\,, J \setminus B_{x,r} = K \setminus B_{x,r}\big\} \le \mathcal{H}^n(L)\,. \tag{0.4}$$

Theorem 2. *Let $H \subset \mathbb{R}^{n+1}$ be closed and $\mathcal{P}(H)$ be a good class. If the minimum in Plateau's problem* (0.1) *is finite, $\{K_j\} \subset \mathcal{P}(H)$ is a minimizing sequence of countably $\mathcal{H}^n$-rectifiable sets and $\mu_j := \mathcal{H}^n \llcorner K_j$, then, up to subsequences, μ_j converges weakly* in $\mathbb{R}^{n+1} \setminus H$ to a measure $\mu = \theta \mathcal{H}^n \llcorner K$, where $K = \mathrm{spt}\,\mu \setminus H$ is a countably $\mathcal{H}^n$-rectifiable set and $\theta \ge 1$. In particular, $\liminf_j \mathcal{H}^n(K_j) \ge \mathcal{H}^n(K)$.*

Although Theorem 2 does not imply in general the existence of a minimizer in $\mathcal{P}(H)$, this might be achieved with little additional work in two interesting cases. The first one is motivated by a very elegant idea of Harrison, which can be explained as follows. Assume that H is a smooth closed compact $n-1$-dimensional submanifold of $\mathbb{R}^{n+1}$: then we say that a relatively closed set $K \subset \mathbb{R}^{n+1} \setminus H$ bounds H if K intersects every smooth curve γ whose linking number with H is 1. A possible formulation of Plateau's problem is then to minimize the Hausdorff measure in this class of sets. Building upon her previous work on differential chains, see [Har12], in [Har14] Harrison gives a general existence result for a suitable weak version of this problem. In the subsequent work [HP13], they prove that the corresponding minimizer yields a closed set K which is a minimizer in the original formulation of the problem. We can recover the theorem of Harrison and Pugh in a relatively short way from Theorem 2. In fact our approach allows one to work, with the same effort, in a more general setting.

Definition 3. *Let $n \geq 2$ and H be a closed set in $\mathbb{R}^{n+1}$. Let us consider the family*

$$\mathcal{C}_H = \left\{\gamma : S^1 \to \mathbb{R}^{n+1} \setminus H : \gamma \text{ is a smooth embedding of } S^1 \text{ into } \mathbb{R}^{n+1}\right\}.$$

One says that $\mathcal{C} \subset \mathcal{C}_H$ is closed by homotopy (with respect to H) if $\mathcal{C}$ contains all elements $\gamma' \in \mathcal{C}_H$ belonging to the same homotopy class $[\gamma] \in \pi_1(\mathbb{R}^{n+1} \setminus H)$ of any $\gamma \in \mathcal{C}$. Given $\mathcal{C} \subset \mathcal{C}_H$ closed by homotopy, we say that a relatively closed subset K of $\mathbb{R}^{n+1} \setminus H$ is a $\mathcal{C}$-filling of H if

$$K \cap \gamma \neq \emptyset \text{ for every } \gamma \in \mathcal{C}. \tag{0.5}$$

We denote by $\mathcal{F}(H,\mathcal{C})$ the family of $\mathcal{C}$-fillings of H.

Theorem 4. *Let $n \geq 2$, H be closed in $\mathbb{R}^{n+1}$ and $\mathcal{C}$ be closed by homotopy with respect to H. Assume that the minimum of the Plateau's problem corresponding to $\mathcal{P}(H) = \mathcal{F}(H,\mathcal{C})$ is finite. Then:*

(a) *$\mathcal{F}(H,\mathcal{C})$ is a good class in the sense of Definition 1.*
(b) *There is a minimizing sequence $\{K_j\} \subset \mathcal{F}(H,\mathcal{C})$ which consists of $\mathcal{H}^n$-rectifiable sets. If K is any set associated to $\{K_j\}$ by Theorem 2, then $K \in \mathcal{F}(H,\mathcal{C})$ and thus K is a minimizer.*
(c) *The set K in (b) is an $(\mathbf{M}, 0, \infty)$-minimal set in the sense of Almgren.*

A second consequence of Theorem 2 is an existence result for the "sliding minimizers" introduced by David, see [Dav14, Dav13].

Definition 5. *Let $H \subset \mathbb{R}^{n+1}$ be closed and $K_0 \subset \mathbb{R}^{n+1} \setminus H$ be relatively closed. We denote by $\Sigma(H)$ the family of Lipschitz maps $\varphi : \mathbb{R}^{n+1} \to \mathbb{R}^{n+1}$ such that there exists a continuous map $\Phi : [0,1] \times \mathbb{R}^{n+1} \to \mathbb{R}^{n+1}$ with $\Phi(1,\cdot) = \varphi$, $\Phi(0,\cdot) = \mathrm{Id}$ and $\Phi(t,H) \subset H$ for every $t \in [0,1]$. We then define*

$$\mathcal{A}(H,K_0) = \left\{K : K = \varphi(K_0) \text{ for some } \varphi \in \Sigma(H)\right\}$$

and say that K_0 is a sliding minimizer if $\mathcal{H}^n(K_0) = \inf\{\mathcal{H}^n(J) : J \in \mathcal{A}(H,K_0)\}$.

Theorem 6. *$\mathcal{A}(H,K_0)$ is a good class in the sense of Definition 1. Moreover, assume that K_0 is bounded, countably $\mathcal{H}^n$-rectifiable with $\mathcal{H}^n(K_0) < \infty$, that $\mathcal{H}^n(H) = 0$, and that for every $\eta > 0$ there exist $\delta > 0$ and $\pi \in \Sigma(H)$ such that*

$$\mathrm{Lip}\,\pi \leq 1 + \eta, \qquad \pi(U_\delta(H)) \subset H. \tag{0.6}$$

Given any minimizing sequence $\{K_j\}$ in the Plateau's problem corresponding to $\mathcal{P}(H) = \mathcal{A}(H,K_0)$, let K be any set associated to $\{K_j\}_j$ by Theorem 2. Then

$$\inf\left\{\mathcal{H}^n(J) : J \in \mathcal{A}(H,K_0)\right\} = \mathcal{H}^n(K) = \inf\left\{\mathcal{H}^n(J) : J \in \mathcal{A}(H,K)\right\}. \tag{0.7}$$

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ACTIVITY REPORT

JAMES M$^{\text{C}}$KERNAN

ABSTRACT. Activity report for Simons Visiting Professor, host Professor Stefan Kebekus, FRIAS.

I spent just over a month visiting the Freiburg Institute for Advanced Studies, from July 21st 2014 to August 24th 2014. The following week I attended a workshop at Oberwolfach, “Komplexe Analysis”.

My host at FRIAS was Professor Stefan Kebekus. The visit was very productive. I worked closely with Professor Chenyang Xu who was also visiting FRIAS over the same period of time. We are writing a paper together with Professor Christopher Hacon, with the title “Boundedness of moduli of varieties of general type”. This paper is close to completion and we will put a version on the arxiv soon and we hope to submit this paper to a journal in the near future. Building on previous work, we show that the moduli functor of canonically polarised varieties is bounded. I gave a lecture on this material at FRIAS on August 19th.

Another collaborator of mine, Paolo Cascini, from Imperial College London, visited me in Freiburg, on July 23rd and 24th. We continued working on a long term project to prove Shokurov’s conjecture on ACC for the log discrepancy. At the moment we are working on a conjecture that if one fixes the dimension then after a bounded number of blows ups we can reduce to the case of abelian quotient singularities. I lectured on this material at the meeting on Complex Analysis I attended at Oberwolfach, which ran from August 24th 2014 to August 29th 2014.

During the last week of my stay in FRIAS, I attended the conference “Complex Analysis and Geometry” at FRIAS. My students Calum Spicer and Roberto Svaldi both visited FRIAS that week. Svaldi and I have a joint project with Professor Morgan Brown and Professor Runpu Zong. Svaldi gave a short presentation of this material at Oberwolfach.

Date: September 4, 2014.

This research stay was partially supported by the Simons Foundation and by Mathematisches Forschungsinstitut Oberwolfach.

We are in the process of writing up this work and we expect to finish writing a paper within a month.

I also had many enoyable mathematical conversations with Stefan Kebekus and some of the other visitors to FRIAS, including Professor Behrouz Taji, who is a post doc at Freiburg University, Professor Steven Lu, and Professor Thomas Szemberg.

Department of Mathematics, University of California, San Diego, 9500 Gilman Drive # 0112, La Jolla, CA 92093-0112, USA

E-mail address: `mckernan@math.ucsd.edu`

Activity Report of Yum-Tong Siu as Simons Visiting Professor at University of Freiburg and Mathematisches Forschungsinstitut Oberwolfach from August 20 to 30, 2014

During the period of August 20 to 30, 2014, as Simons Visiting Professor I first participated in the Conference on Complex Analysis and Geometry in the host university, University of Freiburg, from August 21 to 23, 2014 and gave a talk entitled *Pluricanonical Hodge Decomposition* on August 23, 2014. Then I participated in the Oberwolfach Workshop in Komplexe Analysis (ID 1435) from August 24 to 30, 2014 and gave a talk on August 29, 2014 entitled *Application of Pluricanonical Periods to Problem of Schottky-Jung and Differential Equations.*

The motivation of the both talks is to understand the phenomenon of the deformational invariance of plurigenera in terms of some Hodge decomposition in the pluricanonical setting. The m-genus $\dim_{\mathbb{C}} H^0(X, mK_X)$ of a compact Kähler manifold X is conjectured to remain unchanged when X is holomorphically deformed. The conjecture was proved for the case of projective algebraic X by Siu in 2002. For $m = 1$ such a deformational invariance of $\dim_{\mathbb{C}} H^0(X, K_X)$ for a compact Kähler manifold X is just a direct consequence of the Hodge decomposition. The question is whether the deformational invariance of m-genus for $m \geq 2$ can also be understood in the context of some form of Hodge decomposition with $H^0(X, mK_X)$ as a summand. Both talks discuss the results and the developments in the study of this problem by starting with the simplest case of compact Riemann surfaces and the work of Bol in 1949, Eichler in 1957, Shimura in 1959, and Gunning in 1960.

For a compact Riemann surface X of genus $g \geq 2$, the global coordinate z of the open unit 1-disk Δ as its universal cover can serve as local coordinates of X, giving X a projective structure in the sense that the coordinate transformations are Möbius transformations. Differentiating $(2m-1)$-times an element of $\mathcal{O}_X((1-m)K_X)$ with respect to the global coordinate z of Δ yields an element of $\mathcal{O}_X(mK_X)$. The exact sequence

$$0 \to \operatorname{Ker} d^{2m-1} \to \mathcal{O}_X((1-m)K_X) \xrightarrow{d^{2m-1}} \mathcal{O}_X(mK_X) \to 0$$

yields the exact sequence

$$(*) \qquad 0 \to H^0\left(X, \mathcal{O}_X(mK_X)\right) \xrightarrow{\Theta_m} H^1\left(X, \operatorname{Ker} d^{2m-1}\right) \to H^1\left(X, \mathcal{O}_X((1-m)K_X)\right) \to 0,$$

where $\operatorname{Ker} d^{2m-1}$, consisting of $P(z)(dz)^{1-m}$ with $P(z)$ being a polynomial of degree $\leq 2m-2$, is a flat $\mathbb{C}$-vector bundle over M of rank $2m-1$ when the coefficients of $P(z)$ are used as fiber coordinates. The $(2m-1)$-vector with components $z^k(dz)^{1-m}$ for $0 \leq k \leq 2m-2$ defines a holomorphic section σ_{m-1} of $\left(\operatorname{Ker} d^{2m-1}\right)^* \otimes ((1-m)K_X)$ over X, where $\left(\operatorname{Ker} d^{2m-1}\right)^*$ is the dual bundle of $\operatorname{Ker} d^{2m-1}$. Since $H^1\left(X, \mathcal{O}_X((1-m)K_X)\right)$ is dual to $H^0\left(X, \mathcal{O}_X(mK_X)\right)$, from $(*)$ we have the *pluricanonical Hodge decomposition*

$$(\dagger) \qquad \begin{aligned} H^1\left(X, \operatorname{Ker} d^{2m-1}\right) &= H^0\left(X, \mathcal{O}_X(mK_X)\right) \oplus H^1\left(X, \mathcal{O}_X((1-m)K_X)\right) \\ &\approx H^0\left(X, \mathcal{O}_X(mK_X)\right) \oplus \overline{H^0\left(X, \mathcal{O}_X(mK_X)\right)}, \end{aligned}$$

which becomes the usual Hodge decomposition when $m = 1$ with $\operatorname{Ker} d^{2m-1}$ reduced to the trivial $\mathbb{C}$-line bundle. The map Θ_m is the *m-canonical period map.* The transpose $\Xi_m : H^0\left(X, \mathcal{O}_X(mK_X)\right) \to H^1\left(X, \left(\operatorname{Ker} d^{2m-1}\right)^*\right)$ of the surjective map of $(*)$ is the *dual m-canonical period map* and can be described by the integration of the $\left(\operatorname{Ker} d^{2m-1}\right)^*$-valued 1-form $\sigma_{m-1} f$ over the loops of X for $f \in H^0\left(X, \mathcal{O}_X(mK_X)\right)$. From the second line of $(\dagger)$ the map

$$H^0\left(X, \mathcal{O}_X(mK_X)\right) \times H^0\left(X, \mathcal{O}_X(m'K_X)\right) \to H^0\left(X, \mathcal{O}_X((m+m')K_X)\right)$$

given by mulitplication yields a multiplication formula which produces $(m+m')$-canonical periods from m-canonical periods and m'-canonical periods. This multiplication depends algebraically on X as X varies in the moduli space of all compact Riemann surfaces of genus $g \geq 2$. Since the m-canonical periods satisfy a Riemann relation in the same way as the usual periods (where $m = 1$), the multiplication formula applied m times to the usual periods can be applied to provide new algebraic relations for the usual periods in the Schottky-Jung problem. At this point explicit expressions for the multiplication formula are not yet known.

Schwarz in 1873 used (equivariant) periods of certain compact Riemann surfaces as integral representations for solutions of the Gauss hypergeometric differential equation. Another application of the m-canonical Hodge decomposition is to analogously use (equivariant) m-canonical periods as some generalized form of integral representations for solutions of a wider class of differential equations.

When X is replaced by an n-dimensional compact complex manifold *with projective structure*, we can replace $\operatorname{Ker} d^{2m-1}$ by a flat bundle $\mathcal{F}_{m-1}$ consisting of $P(z_1, \cdots, z_n)\left(dz_1 \wedge \cdots \wedge dz_n\right)^{1-m}$ with $P(z_1, \cdots, z_n)$ being a polynomial of degree $\leq (n+1)(m-1)$ in the local coordinates $z_1, \cdots, z_n$ of the projective structure of X. The analogue of the dual m-canonical period map Ξ_{m-1} can be defined, but no analogue of the m-canonical Hodge decomposition $(*)$ and $(\dagger)$ is known. The decomposition

$$H^n\left(X, \mathcal{F}_{m-1}^*\right) = \bigoplus_{p+q=n} H^q\left(X, \mathcal{O}_X\left(\mathcal{F}_{m-1}^* \otimes \wedge^p T_X^*\right)\right)$$

does not hold even in the case of $m \geq 2$ and $n = 1$, because for any nonzero element $f \in H^0\left(X, \mathcal{O}_X\left((m-1)K_X\right)\right)$ its exterior differential $d\left(f\sigma_{m-1}\right)$ represents a nonzero element of $H^0\left(X, \mathcal{O}_X\left(\mathcal{F}_{m-1}^*\right) \otimes K_X\right)$ which is mapped to 0 in $H^1\left(X, \mathcal{F}_{m-1}^*\right)$. The reason for this phenomenon is that the flat bundle $\mathcal{F}_{m-1}^*$ is not *unitarily flat* in the sense that it cannot carry a positive definite Hermitian metric which is flat in the flat structure of $\mathcal{F}_{m-1}^*$.

In this period, during the conferences in Freiburg and Oberwolfach I myself continued my own research on the conjecture of deformational invariance of plurigenera for compact Kähler manifolds by combining the techniques of Marc Levine of 1983 and 1985 with my old techniques and with some new ones which I just developed. I am now in the last stage of confirming the conjecture by checking the details of my new argument of combining all these techniques.

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Scientific Activity Report

Craig Westerland
University of Minnesota, Twin Cities

September 26, 2014

During the period of 6-14 September 2014, I visited the University of Copenhagen as part of the Simons Visiting Professor program. The following week (14-20 September 2014) I attended the workshop "Topologie" at Oberwolfach. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I would like to thank both institutions for their support.

During my stay at the University of Copenhagen, I worked with Professors Nathalie Wahl and Lars Hesselholt. There were three main topics of discussion:

- Homological stability for Hurwitz spaces, and whether the techniques of my joint work with Ellenberg-Venkatesh [EVW09] could be extended to a larger class of Hurwitz spaces (yielding further extensions of the Cohen-Lenstra heuristics in the function field case).
- The distribution of torsion in the homology of random simplicial complexes (following a conjecture of Matt Kahle) and random triangulated manifolds.
- Whether the probabilistic approach to questions such as the Cohen-Lenstra heuristics could shed light on the Kummer-Vandiver conjecture.

After some discussion, it was felt that the answer to the last question was a simple "no." I will describe the work pursued in the first two subjects below. Conversations on these topics continued during my week in Oberwolfach, and involved other researchers, most notably Oscar Randal-Williams (Cambridge). I also spoke extensively with Chris Schommer-Pries and Nat Stapleton (both at MPI-Bonn) about chromatic homotopy theory.

1 Homological stability for Hurwitz spaces

In [EVW09], our main technical tool is a proof of homological stability for Hurwitz spaces. For a group G, conjugacy class $c < G$, and $n \in \mathbb{Z}_{>0}$, we write $\mathrm{Hur}^c_{G,n}$ for the moduli space of branched covers of $\mathbb{C}$ with n branch points, Galois group G, and monodromy around branch points lying in c. Concretely, there is a forgetful map

$$\mathrm{Hur}^c_{G,n} \to \mathrm{Conf}_n(\mathbb{C})$$

which carries a branched cover to the configuration of its n branch points in $\mathbb{C}$. This is a covering space; the fibre over $\underline{x} = \{x_, \ldots, x_n\}$ is given by the set of coverings with branch locus $\underline{x}$ or, what is the same, the set of homomorphisms $f : \pi_1(\mathbb{C} \setminus \underline{x}) \to G$ (with mondromy in c).

Wahl and I spent a substantial amount of time trying to adapt the usual "arc complex" method of proof of homological stability for configuration spaces to this setting in order to improve the results of [EVW09], but were unsuccessful. The usual surgery arguments do not interact well with the covering data, and so we cannot control both at the same time.

I discussed with Wahl and Randal-WIlliams a new approach to proving stability, based upon a compactification of $\mathrm{Hur}^c_{G,n}$ much akin to the compactification of $\mathrm{Conf}_n(\mathbb{C})$ given by the symmetric product $\mathrm{SP}^n(\mathbb{C})$. I have high hopes for this approach, but it rests upon delicate questions of the nature of the singularities in this compactification. It seems that intersection homology may be useful in this approach.

2 The distribution of torsion in the homology of random simplicial complexes

Kahle [Kah13] has conjectured that the torsion in the homology of a random simplicial complex is Cohen-Lenstra distributed. In forthcoming work, I will give evidence for this conjecture via the following result:

Theorem 1. *There is a probability measure μ on the set $\mathrm{Ch}(\mathbb{Z}_p, r)$ of isomorphism classes of free, finite rank, ungraded chain complexes C_* over $\mathbb{Z}_p$ whose homology $H_*(C_*)$ has torsion free rank r. For a finite abelian p-group H, write $S(H, r)$ for the set of such C_* whose torsion is isomorphic to H. Then*

$$\mu(S(H,r)) = \frac{1}{|H|^r|\,\mathrm{Aut}(H)|}\left(\prod_{j=r+1}^{\infty}(1-p^{-j})\right)$$

This is a precise formulation of a statement wherein the torsion in a finite rank, free chain complex is Cohen-Lenstra distributed. It reduces Kahle's conjecture to:

Conjecture 2. Let $\mathrm{Simp}(r)$ denote the set of isomorphism classes of finite simplicial complexes whose homology has torsion free rank r. Then the map $\mathrm{Simp}(r) \to \mathrm{Ch}(\mathbb{Z}_p; r)$ carrying X to $C^{\Delta}_*(X; \mathbb{Z}_p)$ is well-distributed.

With Wahl, Hesselholt, Ib Madsen (Copenhagen), and Mike Hill (UVA), we discussed whether it is reasonable to expect this conjecture to hold. There is a good argument against it, using the particular (i.e., very far from random) nature of the differential in the simplicial chain complex $C^{\Delta}_*(X; \mathbb{Z}_p)$. There is also a good reason to believe that it is true, as a version of the statement can be proven using random stable CW complexes (and their cellular chain complex). Further questions (discussed with Randal-Williams and Wahl) can be asked regarding the expected torsion in the homology of random triangulated manifolds. I am currently working towards an algebraic model much as the above.

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Scientific Activity Report

Todd Arbogast
Professor in the Department of Mathematics and
Institute for Computational Engineering and Sciences
University of Texas at Austin, USA

This report is for the period September 21 to October 11, 2014. Professor Todd Arbogast attended the Mathematisches Forschungsinstitut Oberwolfach workshop on *Reactive Flows in Deformable, Complex Media.* As a participant of the Simons Visiting Professorship Program, he visited the Technische Universität München, the University of Bergen, and the Eindhoven University of Technology.

1. Oberwolfach Workshop, September 21–26, 2014. The workshop was organized by Professors Margot Gerritsen, Jan Martin Nordbotten, Iuliu Sorin Pop, and Barbara Wohlmuth. Arbogast gave a lecture entitled *Approximation of transport using Eulerian-Lagrangian techniques.*

The talk was well received, and generated an insightful suggestion by Professor Mary Wheeler (also of the University of Texas). Transport of species particles follows the tracelines of the flow, so a species particle at position x at time t^{n+1} came from the point $\check{x}$ at the earlier time t^n. In an Eulerian-Lagrangian method, we use this principle applied to entire volumes of particles. Each element E of a fixed Eulerian computational mesh is filled at time t^{n+1} entirely with particles from $\check{E}$ at t^n. One approximates the concentration c of a chemical species at a time t^n by a constant c_E^n in each element E of a fixed Eulerian computational mesh. The approximation (possibly up to a smaller flux correction) is then

$$c_E^{n+1}|E| = \int_{\check{E}} \mathcal{R}(c^n)\,dx,$$

wherein we us a higher order accurate WENO reconstruction technique $\mathcal{R}$ to approximate the piecewise constant concentrations c^n at time t^n. The difficulty is multiple space dimensions is that the domain of this integration, $\check{E}$, can be a very complicated. We use Strang splitting by dimension to resolve this problem.

Professor Wheeler suggested instead that we should map back to the domain E and apply a quadrature rule to approximate the integral. While this is not a new idea, it is promising for this algorithm for three reasons. First, everything reduces to a finite number of quadrature points. Second, the integral over E requires a Jacobian factor that is generally not treated well. However, I soon realized that one could develop a differential system obeyed by the Jacobian matrix, and therefore an accurate evaluation can be found using a simple ODE solver like Runge-Kutta. Finally, in past work, the integral is not well approximated because low order methods predominate the literature. However, we use high order methods, and so we have very accurate results. We are pursuing the idea to see how well it actually performs in practice.

Many of the lectures related directly to my research and provided inspiration for some of the discussions that followed at the institutions visited after leaving Oberwolfach. Included were two interesting lectures on modeling dynamic capillary pressure and hysteresis given by Professors Majid Hassanizadeh and Ben Schweizer. These ideas were followed up during discussions while visiting München and Eindhoven.

2. München, September 26–October 2, 2014. Professor Barbara Wohlmuth hosted the visit to the Technische Universität München, Garching, Germany. She and I, with her post-docs Drs. Petra Pustejovska and Lorenz John, discussed in some detail the modeling of dynamic capillary pressure and hysteresis. This is essentially a new topic for me, so I was most interested in understanding the mathematical structure of the equations. It is interesting that the dynamic capillary pressure gives added stability to the differential equations, even though inclusion of the effect allows for solutions that are less smooth. I also discussed this subject with post-doc Dr. Elena El Behi-Gornostaeva, who works in another group and specializes in the analysis of the problem for very general hysteresis models. Lorenz and I also discussed generally our individual work on modeling mantle dynamics.

3. Bergen, October 2–7, 2014. Professor Jan Martin Nordbotten hosted the visit to the University of Bergen, Bergen, Norway. Arbogast gave a technical seminar entitled *Approximation of a linear degenerate elliptic equation arising from a two-phase mixture.* The visit to Bergen was very short over the weekend and so included only two working days. Nevertheless, I was able to discuss my work on modeling mantle dynamics and linear degenerate elliptic equations with Professor Florian Radu, who is an expert in this area, and Professor Kundan Kumar, who is trying to model deposition of chemicals and has a set of equations with a similar degeneracy.

I also had the chance to discuss with Professor Ivar Aavatsmark the approximation of elliptic operators on quadrilateral (2D) and hexahedral (3D) computational meshes using finite volume and mixed finite element techniques. I discussed a new approach that I am taking, and I got some interesting feedback, especially related to the importance of preserving the maximum principle of the differential equation.

4. Eindhoven, October 7–11, 2014. Professor Iuliu Sorin Pop hosted the visit to the Eindhoven University of Technology, Eindhoven, Netherlands. Arbogast gave a lecture entitled *Multiscale mixed methods for heterogeneous elliptic problems,* at the Eindhoven Multiscale Institute. The talk was well received, and was a focus of outreach to Eindhoven honors students, many of whom also studied in some detail two of my papers on the subject of my talk.

The visit to Eindhoven was perhaps the culmination of my discussions on flow in porous media. With Professor Pop and his group, we discussed their approach to modeling elliptic flow problems using finite volume methods on quadrilaterals and hexahedra. I also learned about the total flux method from Professor J.H.M. ten Thije Boonkkamp. Unlike my approach, in his approach one includes the diffusive flux within the calculation of the transport, which has the advantage that operator splitting is not needed to compute the diffusive contribution in advection-diffusion equations. Perhaps this will influence my future research in this area.

Most of my time with Professor Pop was spent considering in detail the equations of two-phase flow in porous media with dynamic capillary pressure and hysteresis. I learned important properties of the system (that it is pseudo-parabolic) and the underlying physics of the system. Although the system can be derived from the point of view of thermodynamics (as Professor Majid Hassanizadeh pointed out during the workshop), the coefficients are not readily obtained. We considered a Leverette J-function model, which is that the capillary pressure is

$$p_{\mathrm{c}}(s) = \sigma \cos(\theta) \sqrt{\frac{\phi}{K}} J(s),$$

where σ is the surface tension, θ is the contact angle, ϕ and K are the porosity and absolute permeability of the rock, s is the saturation, and J is a universal function. We recognize that the contact angle is dynamic, $\theta(t)$, and we added a term representing a hysteretic jump in the saturation value between imbibition and drainage, $J = J(s - \gamma\,\mathrm{sign}(s_t))$. Linearization results in the standard capillary model with dynamic capillary pressure and hysteresis. However, we have a set of measurable quantities that define the parameters in terms of $\theta(0)$, $\theta'(0)$, and γ, which depends on the geometry. We need to see if this result matches experimental measurements. If so, we have a macro-scale justification of the equations and definition of the parameters from basic and measurable parameters.

5. Acknowledgment. This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Scientific Activity Report
ZHI-MING MA

Period

19 October - 25 October, 2014, at Oberwolfach Institute;
25 October - 8 November, 2014, at Bielefeld University.

Scientific Activity at Oberwolfach Institute

At Oberwolfach Institute I participated in the workshop "Dirichlet Form Theory and its Applications" (code 1443).

I was the first speaker of the workshop. I presented a talk with the title "Some resent results on quasi-regular semi-Dirichlet forms". See below for an abstract of my talk.

During the workshop we had many interesting and stimulating discussions. Since the celebrated result of M. Fukushima on the connection between regular Dirichlet forms and nice Markov processes in 1971, the theory of Dirichlet forms has been rapidly developed and has brought a wide range of applications in various related areas of mathematics and physics. We believe that this Oberwolfach workshop is very important and will enhance further the the future development of the theory of Dirichlet forms and its applications.

I chaired the last session of the workshop and gave a conclude remark at the end of the workshop. On behalf of the participants we thank the organizers and the Oberwolfach Institute, including the financial support from the Simons Foundation, for providing us this wonderful opportunity to communicate our scientific research ideas and to exchange our experiences. The workshop will have a strong influence in future development of the theory of Dirichlet forms and applications.

Scientific Activity at Bielefeld University

On Wednesday, October 29. I gave a talk in the Department of Mathematics, The title of my talk is"A tansform of Markov Jump processes and applications in genetic study". An abstract of my talk is at the end of this report.

On Wednesday Nov. 5, 2014, together with Prof. Roeckner, we met Professor Dr. Sagerer, the Rektor of Bielefeld University. We had a pleasant conversation. During the conversation we recalled our long standing collaboration between Bielefeld University and The Chinese Academy of Sciences since 1987, at that time I came to Germany for the first time as a Humboldt Research fellow. And we looked forward the future development between our two institutions.

On Wednesday Nov. 5, 2014, Prof. Roeckner and myself had a meeting with some professors in Bielefeld, including Professors Goetze,Grigorian,Riedel and others. We had a preliminary discussion about the possibility and how can we establish a Collaborative Research Centre (CRC) between scientists in Germany and in China.

On Thursday Nov. 6, 2014, Roeckner and myself met another group of scientists and we discussed again about the possibility and how can we establish a Collaborative Research Centre (CRC) between scientists in Germany and in China.

Some Resent Results on Quasi-regular Semi-Dirichlet Forms

Abstract

In this talk we present some results on quasi-regular semi-Dirichlet forms. In particular, we present our recent results (joint with Wei Sun and Li-Fei Wang) on Fukushima type decomposition for semi-Dirichlet forms and discuss some related topics. Under a reasonable assumption, we obtain the following result.

Theorem

Suppose that $(\mathcal{E}, D(\mathcal{E}))$ is a quasi-regular semi-Dirichlet form on $L^2(E;m)$ satisfying the Assumption. Then for $u \in D(\mathcal{E})_{loc}$ the following two assertions are equivalent to each other.

(i) u admits a Fukushima type decomposition. That is, there exist $M^{[u]} \in \mathcal{M}_{loc}^{I(\zeta)}$ and $N^{[u]} \in \mathcal{L}_c$ such that

$$(1) \qquad \tilde{u}(X_t) - \tilde{u}(X_0) = M_t^{[u]} + N_t^{[u]}, \quad t \geq 0, \quad P_x\text{-}a.s. \ for \ \mathcal{E}\text{-}q.e. \ x \in E.$$

(ii) u satisfies Condition (S) specified below.

$$(S): \quad \mu_u(dx) := \int_E (\tilde{u}(x) - \tilde{u}(y))^2 J(dy, dx) \text{ is a smooth measure.}$$

Moreover, if u satisfies Condition (S), then the decomposition (1) is unique up to the equivalence of local AFs, and the continuous part of $M^{[u]}$ belongs to $\dot{\mathcal{M}}_{loc}$.

A Tansform of Markov Jump Processes
and Applications in Genetic Study

Abstract

Recently in the study of genetic coalescent with recombination, we encountered an interesting example of Markov jump process with continuous state space. For the purpose of our study, apart from making use of some existing results, we need also to develop some new results concerning Markov jump processes. In particular, we need to investigate the behaviour of Markov jump processes under transformations of state spaces. The latter by its own is also of importance in the theory of Markov processes. In this talk I shall report our results (joint with X.Chen et.al.) in this research direction.

Acknowledgement

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Address:

Prof. Zhi-Ming Ma, Academy of Mathematics and Systems Science, CAS, Zhong-guan-cun East Road 55, Beijing 100190, China. Email: mazm@amt.ac.cn Date: Nov.20, 2014.

Scientific Activity Report
Simons Visiting Professorship: November 4–12, 2014
Kiran S. Kedlaya (University of California, San Diego)

This SVP was associated to the workshop "Valuation theory and its applications" held October 27–31, 2014 at MFO. The period of the SVP was spent in the following locations, and included the following activities. (The missing dates November 1–3 were devoted to personal travel.)

- **November 4-6**: Université de Montpellier, visiting Andrea Pulita. I gave an algebraic geometry seminar on November 5 entitled "Formal structure of flat connections". In addition, I collaborated with Pulita on the subject of connections on p-adic analytic curves, especially convergence polygons and the index formula for de Rham cohomology. We also prepared for his HDR defense, which was originally scheduled to take place during this visit but had to be postponed due to an administrative mixup.
- **November 7**: École Polytechnique Federale de Lausanne, visiting Philippe Michel.
- **November 8–12**: University of Cambridge, visiting Gabriel Paternain and Mark Gross. I gave a colloquium lecture on November 11 as follows:
  - Title: A brief (pre)history of perfectoid spaces
  - Abstract: Classical Hodge theory can be interpreted as the relationship between the topology of manifolds and the cohomology of differential forms. In arithmetic geometry, the parallel subject of p-adic Hodge theory relates Galois actions on etale cohomology to differential forms. Recently, this subject has been overturned by a series of developments culminating in the construction of a class of objects called "perfectoid spaces". Without trying to give too many formal definitions, I will indicate some of the key ideas that go into the theory; as time permits, I'll also mention some of the directions in which the work of Scholze carries this theory far beyond its point of origin.

This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. Additional support was provided by the visited universities.

At this time, there are no publications or preprints derived directly from this visit, though I expect to produce later a research paper on the index formula for p-adic connections in collaboration with Andrea Pulita and Jérôme Poineau (Strasbourg). In addition, I am planning to prepare a survey article on p-adic connections for the upcoming Simons Symposium on non-archimedean and tropical geometry (February 2015), and as the collobaration with Pulita is germane to this topic, support from this visit will be acknowledged therein.

Scientific Activity Report (Simons Visiting Professorship)

HSIEN-KUEI HWANG

Before attending the workshop "Probability, Trees and Algorithms" (organized by Luc Devroye and Ralph Neininger, November 2–8, 2014), I spent five days (October 28–November 1) visiting the Institut für Mathematik, J. W. Goethe-Universität (Frankfurt am Main). This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach. I was working with Ralph Neininger on two major topics: *dependence of random m-ary search trees* and the *profiles of random digital search trees*. We made significant progresses on these topics during my stay, and the two papers containing our new findings are now in preparation. In particular, the m-ary search tree paper is expected to be finished soon (in a month or so). We summarize briefly our results below, which were also presented during the Workshop (November 6, 2014) jointly with Ralph Neininger.

The m-ary search tree is a class of data structures introduced by Muntz and Uzgalis [5] in 1971 in computer algorithms to support efficient searching and sorting of data. When constructed from a random permutation of n elements, the space requirement of such a *random m-ary search tree* ($m \geqslant 3$) is known to exhibit a *phase change phenomenon*: its distribution tends to Gaussian for large n when the branching factor m satisfies $3 \leqslant m \leqslant 26$ but does not approach a fixed limie law when $m \geqslant 27$; see [4, 1, 3] and the references therein. More precise approximation results are also known; see [3, 2]. This phase change has attracted the attention of many probabilists since the publication of [1] and has been widely approached in the last decade. In addition to the method of moments and contraction method, other approaches used include urn models, martingales, renewal theory, etc.

On the other hand, what is also known is that the total path length T_n (the sum of the distance between each node to the root) does not change its limiting behavior and tends asymptotically, after properly centered and normalized, to a limit law.

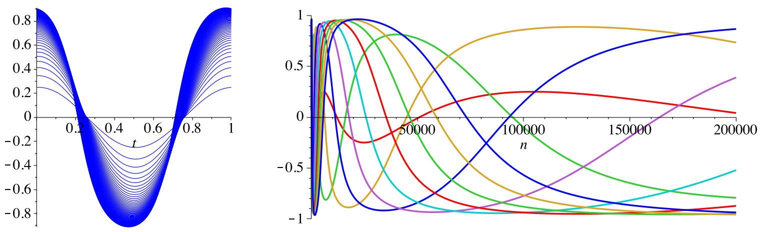

Figure 1: *The periodic functions of $F_\rho(t)$ for $m = 27, \ldots, 100$ (left) and $F_\rho(c \log n)$ for $m = 27, 54, \ldots, 270$ (right).*

The new question we addressed was "to which extent does T_n depend on S_n"? To our surprise, the random variables T_n, despite the strong dependence of its definition on S_n, turn out to be asymptotically independent of S_n when $3 \leqslant m \leqslant 26$, and dependent when $m \geqslant 27$. More precisely, the correlation coefficient satisfies

$$\boxed{\rho(S_n, T_n) \sim \begin{cases} 0, & \text{if } 3 \leqslant m \leqslant 26; \\ F_\rho(c \log n), & \text{if } m \geqslant 27, \end{cases}}$$

where $F_\rho(t)$ is a 1-periodic function and c is a structural constant.

While one might ascribe this counter-intuitive result to the possibly nonlinear correlation between T_n and S_n, we enhance such an asymptotically uncorrelated phenomenon by a stronger joint limit law, which again puts an accent on the asymptotic independence between T_n and S_n when $3 \leqslant m \leqslant 26$. For larger m, they are asymptotically dependent and we have a precise characterization of the joint limit law. The tools we need are adapted from [6, 7].

Moreover, replacing the total node path length by total key path length (summing over all keys instead of over all nodes) does not change the major phenomena. Indeed, the same types of results hold in a more general setting.

Finally, the consideration of random m-ary search trees can be extended to other random trees of logarithmic height such as quadtrees, fringe-balanced binary search trees, etc., and the same phenomena hold. Details of all these results will be contained in a forthcoming paper jointly written with Hua-Huai Chern, Michael Fuchs and the two authors of this abstract.

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SAN DIEGO STATE UNIVERSITY
Department of Mathematics and Statistics
San Diego State University
5500 Campanile Drive
San Diego, CA 92182-7720

December 31, 2014

Simons Visitor Activity Report
Chris Rasmussen

The following is a summary of my activity at Oberwolfach (December 7-13, 2014) and research stay with Professor Reinhard Hochmuth as a Simons Visiting Professor (December 15-20, 2014).

At the Oberwolfach meeting I gave a presentation that outlined a theoretical/methodological approach for coordinating individual and collective level mathematical progress as it occurs in situ. The presentation was followed by a lively and thoughtful discussion with approximately one-third of the meeting participants. The week was filled with attending many other talks, in-depth discussions with presenters, forming new collaborations, and discussing opportunities for publications related to the various presentations.

My stay at Leibniz University began with an orientation meeting with Dr. Hochmuth and his doctoral students in which the doctoral students told me a bit about their research interests and the various research projects that they each were working on. I then met individually throughout the week with doctoral students to discuss in more depth about their research, the theoretical perspectives that were informing their work, and the methodologies they were using.

I also held a half-day workshop on the methodology for documenting collective activity. Collective activity is defined to be a classroom's normative ways of reasoning and offers a rigorous empirical approach for documenting how mathematical truths are established through argumentation. Central to the methodology is a systematic use of Toulmin's argumentation scheme, which describes the structure and function of an argument in terms of four parts: the data, the claim, the warrant, and the backing. In the workshop I provided a theoretical overview for the method, reviewed the approach, and then provided sample classroom transcripts that allowed attendees to practice the approach.

On a different occasion with the group we discussed the methodology of design based research (DBR). I shared with the group my experiences and insights from several years of DBR that my research team and I have conducted in differential equations and linear algebra. I also shared with the group a paper on DBR by Cobb and Gravemeijer (two pioneers in this method) and we had a lengthy discussion about this reading.

Another paper that we discussed at length was a recent review of the last ten years of research in undergraduate mathematics education that I my colleague conducted for an upcoming handbook to be published by the National Council of Teachers of

Mathematics. This chapter provides a comprehensive review of the advances in undergraduate mathematics education research and outlines several areas for future growth. I also was learned more about the projects led by Professor Reinhardt by attending project meetings and meeting with his extended team.

Lastly, I gave a colloquium that discussed insights from a large, national study of post-secondary Calculus I programs in the US. In this talk I presented results from a national survey of students and faculty regarding which students choose to switch out of a science, technology, engineering, or mathematics (STEM) major and their in class and out of class experiences that contribute to their decisions to leave a STEM major. I also presented findings from case study analyses at five exemplary calculus programs at US institutions that offer a doctoral degree in mathematics and the seven different programmatic and structural features that are common across the five institutions.

Acknowledgement: This research stay was partially supported by the Simons Foundation and by the Mathematisches Forschungsinstitut Oberwolfach.

Sincerely,

Chris Rasmussen
Professor, Department of Mathematics and Statistics
San Diego State University